The role of endogenous capital depreciation rate in Dynamic Stochastic General Equilibrium models: Evidence from Canada

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Abstract

This paper investigates the optimal behavior of the main real macroeconomic variables in a Dynamic Stochastic General Equilibrium (DSGE) framework augmented with a time-varying depreciation rate of capital stock and an endogenous production of maintenance goods. For this purpose I explicitly define a depreciation rate function which is positively related to the utilization rate of capital and inversely related to the ratio of maintenance to capital stock. Along the balanced growth path, the trend of the depreciation rate is driven by the steady state value of the investment-specific technology progress (IST). The Bayesian estimation exercises performed on the Canadian economy show that, in response to a positive shock on marginal efficiency of investment (MEI) which drives the economic business cycle, the model is able to generate co-movement in all the main real endogenous variables including consumption, maintenance and depreciation. The optimal paths are amplified with respect to the baseline model with a constant depreciation and no maintenance costs, and their convergence dynamics are delayed as a consequence of acceleration in depreciation through the obsolescence effect. The model also shows that, in response to a positive IST shock both depreciation and maintenance decline due to an increase in the average service life of existing capital. Finally, I include in the model a shock which affects the transformation process of final goods into maintenance goods, named the maintenance-specific technology progress (MST). In the short run, this shock is the key-driver of the growth in real maintenance.

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1 Introduction

In the macroeconomic modeling framework, in general, it is largely assumed that over time capital stock depreciates at a constant rate. This assumption is supported by an extensive literature in the related field[1]. Several microeconomic studies, however, argue that especially during the periods of intensive technological progress depreciation fluctuations can not be considered negligible[2]. The evidence shows, in fact, that over the past decades depreciation rate of capital has accelerated and it has grown persistently especially in the high-tech sectors. Fig.1 depicts the ri-elaborated series for depreciation rate and the relative price of investment (NIPA), taken from the Bureau of Economic Analysis (BEA), which publishes prices unadjusted for quality and thus implicitly includes the effects of obsolescence on capital stock. The figure shows that in the U.S., especially after the second half of the 1960s, capital depreciation rate has grown consistently. This surge is accompanied by a fall in the NIPA's relative price of investment, and the historical period, indeed, is characterized by a strong economic boom and widespread of new technological goods. As Keynes (1936) argues, when a new investment good becomes more efficient more capital with higher qualitative characteristics is produced, this induces a more intensive use of the already installed capital and, hence, an acceleration in its depreciation rate. Therefore, especially during the periods of intensive technological progress accounting for depreciation due to wear and tear, deterioration and obsolescence may improve the estimations of the aggregate capital stock of the economy. All these components of depreciation can be influenced by innovation, usage, aging and maintenance.

In the existing macroeconomic literature it is commonly assumed that depreciation is positively related to maintenance expenses and negatively to capital utilization rate[3]. The main contributions come from, for example, Boucekkine and Ruiz-Tamarit (2003) who argue that depreciation, maintenance and utilization are pro-cyclical in response to a neutral technology shock when the sensitivity of depreciation rate function is higher for capital utilization than for maintenance. They are countercyclical otherwise. Boucekkine et al. (2010) assuming endogenous scrapping time in a vintage AK model find that the scrapping rate of capital is pro-cyclical in response to a neutral technology progress, which induces maintenance costs to co-move in short run, while use-dependent depreciation and obsolescence are both countercyclical. Boucekkine et al. (2009) propose a two-sector vintage capital model with neutral and investment specific technology (IST) shocks. According to their model, both the use-related and age-related components of depreciation are pro-cyclical in response to the IST shock, implying co-movement in the economic depreciation rate. On the contrary, the response to a neutral

[1] See, for example, Smets and Wouters (2007), Justiniano and Primiceri (2008), Justiniano et al. (2010, 2011) for DSGE models with constant depreciation rates. And Epstein and Denny (1980), Hulten and Wykoff (981a,b), Nadiri and Prucha (1996), Jorgenson (1996), Oliner (1996), Huang and Diewert (2011) for the estimations of the depreciation rates.

[2] See, for example, Tevlin and Whelan (2003), Doms et al. (2004), Geske et al. (2007) and Angelopoulou and Kalyvitis (2012).

[3] See the seminal works of Greenwood et al. (1988) and McGrattan and Schmitz (1999).

technology shock is null for the use-related depreciation, and countercyclical for the age-related and, hence, for the economic depreciation rates. Albonico et al. (2014), on the other hand, find that maintenance costs and capital depreciation rate co-move in response to a total factor productivity shock, while maintenance is countercyclical in response to the IST shock.

Using the decentralized DSGE model by Justiniano et al. (2011) as baseline I extend it incorporating a new sector where perfectly competitive firms use a fraction of final goods in order to produce maintenance goods and services, which are purchased by households who detain the capital stock. I assume, in addition, that capital depreciation vary over time. Following the existing literature, depreciation is negatively related to maintenance expenses and positively related to capital utilization rate. In each sector agents face a sector-specific optimization problem and there are four main technology shocks that hit the economy: the neutral labor augmenting technology, the investment specific technology (IST), the maintenance specific technology (MST), and the marginal efficiency of investment(MEI).

I estimate both the maintenance model and the baseline model of Justiniano et al. (2011) following the Bayesian approach on Canadian economy over 1981Q2 - 2015Q1. The baseline model perfectly replicates the qualitative results in Justiniano et al. (2011) model, which was estimated on the U.S. data.

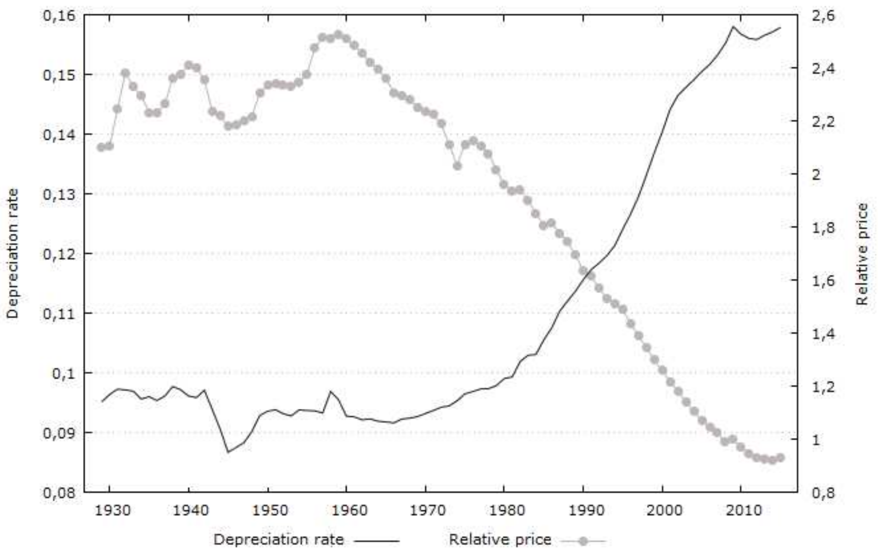
Overall, I argue that the maintenance model behaves fairly better with respect to the baseline model in replicating the Canadian economy. From the comparison of the posterior second moments and correlations of the real endogenous variables it emerges that the maintenance model mimics the respective actual statistics better than the baseline model.

Including maintenance expenses and endogenous capital depreciation rate in a DSGE model helps to generate co-movement in all the main endogenous variables. In fact, I find that real consumption, contrarily to Justiniano et al. (2011), together with the other main macroeconomic variables (investment, hours worked, real wages, inflation, nominal interest rate, rental rate of capital, effective capital, utilization rate, and depreciation and maintenance) are all pro-cyclical in response to a positive MEI shock, as well as to the neutral labor augmenting technology shock. The MEI shock is the main driver of the economic business cycle. It explains around 61% of variability in output in the maintenance model and around 21% in the baseline model. The MEI shock also explains more than 90% of variation in both the depreciation rate and real maintenance in long run. The effect of the IST shock is confirmed to be negligible and in the maintenance model its explanatory power further declines, 0.30% with respect to 2.36% in the baseline model. The labor augmenting technology progress explains 14.34% and 3.49% of output growth in the two models, respectively. The MST shock is found to be significant in explaining the variability in real maintenance growth in the short run.

According to the estimation results, both maintenance and depreciation are pro-cyclical in response to all the technology shocks. In the case of a positive MEI shock depreciation rate accelerates due to obsolescence effect on capital stock. Maintenance increases in long run due to a more intensive use of the existing capital and to its renovation or updating. On the contrary, both depreciation and maintenance decline when a positive IST shock occurs because of an increase in the cumulated optimal capital lifetime. Finally, the maintenance model confirms the literature findings according to which capital depreciation is more volatile than output. Capital stock is estimated to depreciate 14% annually on average. Maintenance and investment are found to behave as complements in response to the MEI shock and as gross substitutes in response to IST shock in short run.

In section two it is described the analytical structure of the maintenance model. The model is log-linearized around the steady state and the linear rational expectations equations are used in the estimation analysis. In section three are presented the results from posterior estimation.

Fig. 1: U.S. Depreciation rate and relative price of private nonresidential equipment and software (1929-2015)



Source: BEA estimations. Author's calculations.

2 Analytical framework

I expand the model by Justiniano et al. (2011) introducing a new sector in which perfectly competitive firms produce maintenance goods and services. I assume this transformation process is subject to maintenance adjustment costs and it is affected by the maintenance-specific technology (MST) progress. Furthermore, I assume that capital depreciation is determined endogenously to the model. Following the depreciation-in-use assumption, according to which when capital is used with higher intensity in the production process depreciation accelerates, I assume it to be positively related to the capital utilization rate. On the contrary, depreciation can be slowed down through the maintenance and repair activities therefore, I assume depreciation and maintenance to be negatively related[4].

In a decentralized model setting, agents in each sector deal with a sector-specific optimization problem[5]. I assume that the optimal level of maintenance expenditures is controlled by the households who detain all the amount of capital stock. Hence, beyond consumption and new capital, they may decide to purchase maintenance goods in order to repair or renovate depreciated capital because of wear and tear (usage), aging, accidental damages or quality innovations (obsolescence), demanding maintenance goods and services to the perfectly competitive maintenance-goods producers. Final goods are produced in a perfectly competitive environment through a combination of intermediate goods and are used as inputs to consumption, maintenance and investment. Firms in a perfectly competitive maintenance sector acquire units of final goods and, given positive maintenance adjustment costs, transform them into efficiency units of maintenance that are ready to "replace" the depreciated capital. New investment goods are produced with a similar mechanism, subject to positive investment adjustment costs, and are sold to perfectly competitive capital-goods producers. The latter ones transform new investment into new capital which is finally sold to households. Households transform new capital in installed capital which is then rent to the intermediate-goods producers as effective capital. The intermediate-goods producers operate in monopolistic competition. A fraction of firms is allowed to optimize for the level of price following Calvo pricing approach. Households, among others, choose the amounts of government bonds holdings. Each household is a monopolistic supplier of specialized labor, that is aggregated by employment agencies into homogeneous labor and which is then sold to intermediate good producers for their production process. Government implements a Ricardian fiscal policy and the monetary authority sets the nominal interest rate according to a Taylor interest rate rule.

In this framework the equilibrium capital depreciation rate follows a growing trend that, along the balanced growth path, depends on the steady state level of the IST progress. This is in line with the related literature according to which the rate of depreciation is inversely related to the relative price of new investment.[6] Indeed, the economic depreciation rate is defined by the National Account Systems as a decline in the asset value and is generally estimated according to the methodology of used-asset prices. This suggests that the lower the price of the used asset the higher the related depreciation. Furthermore, it is shown that in a perfectly competitive market the quality-unadjusted relative price of investment equals the inverse of disembodied[7]

[4] For similar implementations of the depreciation rate function in the macroeconomic framework see Albonico et al. (2014), Boucekkine and Ruiz-Tamarit (2003), and Licandro and Puch (2000).

[5] For a complete scheme of the model please refer to Fig.**??** in the Appendix.

[6] See, among others, Diewert and Schreyer (2006) and Boucekkine et al. (2009).

[7] As it is stated in, for example, Greenwood et al. (1997) and Boucekkine et al. (2009), the disembodied investment-specific technology progress aims to reduce the marginal cost of production of one extra unit of investment. On the other hand, technology progress is said to be embodied when it contributes to increase

investment-specific technology progress. The negative correlation between the relative price of investment and depreciation is also evident in Fig.1.

Given that the prices are unadjusted for quality, I interpret the IST shock, which affects the new investment goods producers, as the disembodied investment-specific technology progress. This shock determines the physical deterioration of capital stock. The MEI shock in the model affects the transformation process of new investment goods into installed capital and, in the final equilibrium set up, together with the IST progress it propagates the aggregated effects into real economy through the capital accumulation process[8]. I interpret the MEI shock as the embodied investment-specific technology progress as it explains the obsolescence component of the depreciation rate.

Below it is described the analytical structure of the model.[9]

The final good sector

Perfectly competitive firms combine a continuum of intermediate goods $\{Y_t(i)\}_i$, $i \in [0,1]$, in order to produce final good Y_t, given the Dixit and Stiglitz (1977) CES aggregate technology. Their profit maximization problem is, therefore

$$\begin{aligned} \max_{Y_t, Y_t(i)} \quad & P_t Y_t - \int_0^1 P_t(i) Y_t(i) di \\ s.t. \quad & Y_t = \left[\int_0^1 Y_t(i)^{\frac{1}{1+\lambda_{p,t}}} di \right]^{1+\lambda_{p,t}} \end{aligned} \tag{P1}$$

where $\lambda_{p,t}$ is the price mark-up shock following an exogenous stochastic ARMA(1,1) process, which, as stated in Justiniano et al. (2011), helps to capture the highly volatile inflation patterns

$$\log \lambda_{p,t} = (1-\rho_p) \log \lambda_p + \rho_p \log \lambda_{p,t-1} + \varepsilon_{p,t} - \theta_p \varepsilon_{p,t-1}, \qquad \varepsilon_{p,t} \sim i.i.d.N(0, \sigma_p^2) \tag{2.1}$$

Final goods are purchased by the households for consumption purposes, by the investment-goods producers, who transform them into efficiency units of new investment, and by the maintenance-goods producers, who transform them into efficiency units of capital maintenance.

The intermediate good sector

In this sector firms are assumed to operate in a monopolistic regime and each one produces a diversified intermediate good by combining the amounts of effective capital $K_t(i)$ and effective labor $L_t(i)$, according to a Cobb-Douglas technology. The profit maximization problem is as follows

productivity through a quality improvement of new investments.

[8] In fact, when reducing the model to a one sector representation, both these shocks enter linearly into the law of motion of capital stock, this implies that they are different elements of one composite total investment shock. The last section of Appendix B describes the setting-out of the respective one sector model.

[9] For a complete description of the model and the derivation of the optimal conditions please refer to Appendix B.

$$\max_{L_t(i),K_t(i)} \quad P_t(i)Y_t(i) - W_t L_t(i) - R_t^k K_t(i)$$
$$s.t. \quad Y_t(i) = A_t^{1-\alpha} K_t(i)^\alpha L_t(i)^{1-\alpha} - A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}} F \tag{P2}$$

where W_t is the aggregate level of nominal wages and R_t^k is the nominal return on capital. A_t is a non-stationary process representing labor-augmenting technology shock. Its growth rate, $\Delta \log A_t = z_t$, follows the following stationary AR(1) process

$$z_t = (1-\rho_z)\gamma_z + \rho_z z_{t-1} + \varepsilon_{z,t}, \qquad \varepsilon_{z,t} \sim i.i.d.N(0, \sigma_z^2) \tag{2.2}$$

Similarly, Υ_t represents a non-stationary process for the investment-specific technology progress. Its growth rate, $\Delta \log \Upsilon_t = v_t$, follows the following stationary AR(1) process

$$v_t = (1-\rho_v)\gamma_v + \rho_v v_{t-1} + \varepsilon_{v,t}, \qquad \varepsilon_{v,t} \sim i.i.d.N(0, \sigma_v^2) \tag{2.3}$$

Finally, F represents the fixed costs. Its value is chosen such that profits are zero in steady state and it is multiplied by the composite technology factor $A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}}$ in order to guarantee the existence of a balanced growth path. When fixed costs are high relative to the production capacity, given by the combination of capital, labor factors and neutral technology, an intermediate goods producer i is constrained to exit the market and the production output, $Y_t(i)$, is null.

The price optimization process is set according Calvo (1983). Every period a fraction ξ_p of intermediate firms resets its prices according to the following indexation rule

$$P_t(i) = P_{t-1}(i)\pi_{t-1}^{\iota_p}\pi^{1-\iota_p} \tag{2.4}$$

where π_t represents the gross inflation and π its steady state level. The fraction ξ_p, thus, represents the natural level of price stickiness. The remaining fraction of firms, $1-\xi_p$, is able to optimize for the price level $\tilde{P}_t(i)$ the present discounted value of future profits subject to the optimal intermediate goods demand function, that is

$$\max_{\tilde{P}_t(i)} \quad E_t \sum_{t=0}^{\infty} \xi_p^s \beta^s \frac{\Lambda_{t+s}}{\Lambda_t} \left[\tilde{P}_t(i)\pi_{t,t+s} - MC_{t+s}\right] Y_{t+s}(i)$$
$$s.t. \quad Y_{t+s}(i) = \left[\frac{\tilde{P}_t(i)}{P_{t+s}}\pi_{t,t+s}\right]^{-\frac{1+\lambda_{p,t+s}}{\lambda_{p,t+s}}} Y_{t+s} \tag{P3}$$

where $\pi_{t,t+s} = \prod_{j=0}^{s} \pi_{t+j-1}^{\iota_p}\pi^{1-\iota_p}$. The term MC_t represents nominal marginal cost and is substituted for the average variable cost, while Λ_t is the marginal utility of nominal income of the representative household who owns the firm.

The maintenance goods sector

I assume that perfectly competitive firms purchase units of final good, Y_t^m, in order to transform them into maintenance goods or services, M_t, which are then sold to households at the unit price of maintenance P_t^m. The transformation process from Y_t^m to M_t incurs positive maintenance adjustment costs given by $f(Y_t^m/Y_{t-1}^m)$ that satisfies $f = f' = 0$ and $f'' > 0$ in steady state.[10] Given the functional form of $f(\cdot)$, an increase in the amount of final good designated for maintenance today, reduces the expected adjustment cost of maintenance.

Firms maximize the expected discounted value of future profits with respect to Y_t^m and M_t subject to a technology that transforms efficiency units of final goods into efficiency units of maintenance goods, as follows

$$\begin{aligned} \max_{M_t, Y_t^m} \quad & E_t \sum_{s=0}^{\infty} \beta^s \Lambda_{t+s} \left[P_{t+s}^m M_{t+s} - P_{t+s} Y_{t+s}^m \right] \\ s.t. \quad & M_{t+s} = d_{t+s} \left[1 - f\left(\frac{Y_{t+s}^m}{Y_{t+s-1}^m} \right) \right] Y_{t+s}^m \end{aligned} \tag{P4}$$

where d_t is the maintenance specific technology shock (MST) described by an AR(1) exogenous stochastic process

$$\log d_t = \rho_d \log d_{t-1} + \varepsilon_{d,t}, \qquad \varepsilon_{d,t} \sim i.i.d.N(0, \sigma_d^2) \tag{2.5}$$

Calling $\tilde{M}_t = (P_t^m/P_t)M_t$ real maintenance in consumption units and combining it with the zero profit condition of the firms yields $Y_t^m = (P_t^m/P_t)M_t$. So, the fraction of final good that is used as input in the production of maintenance goods equates real maintenance in consumption units, i.e. $Y_t^m = \tilde{M}_t$.

The first order condition with respect to the efficiency units of maintenance, M_t, defines the equilibrium level of maintenance price as the shadow value of maintenance goods Γ_t relative to the shadow value of consumption Λ_t

$$P_t^m = \frac{\Gamma_t}{\Lambda_t} \tag{2.6}$$

The first order condition of the optimization problem with respect to Y_t^m establishes the optimal supply of maintenance services, which depends on the maintenance adjustment technology and on the maintenance specific technology shock, as follows

$$\Lambda_t P_t = \Lambda_t P_t^m d_t \left[1 - f\left(\frac{Y_t^m}{Y_{t-1}^m} \right) - \frac{Y_t^m}{Y_{t-1}^m} f'\left(\frac{Y_t^m}{Y_{t-1}^m} \right) \right] + \beta E_t \Big\{ \Lambda_{t+1} P_{t+1}^m d_{t+1} \left(\frac{Y_{t+1}^m}{Y_t^m} \right)^2 f'\left(\frac{Y_{t+1}^m}{Y_t^m} \right) \Big\} \tag{2.7}$$

When maintenance adjustment costs are zero, i.e. $f = f' = 0$, the relative price of maintenance with respect to consumption equals the inverse of the maintenance specific technology shock, that is $P_t^m/P_t = d_t^{-1}$.

The investment good sector

[10] The functional form for the maintenance adjustment costs and the underlying assumptions have been set following Christiano et al. (2005) and Justiniano et al. (2011)

A fraction of final good Y_t^I is purchased by perfectly competitive investment-goods producers in order to transform it into investment goods I_t expressed in efficiency units, which are further sold to the capital-goods producers at the unit price P_t^I. These firms maximize their profit function subject to a production technology which accounts for the investment-specific technology progress, Υ_t, as follows

$$\begin{aligned} \max_{I_t, Y_t^I} \quad & P_t^I I_t - P_t Y_t^I \\ s.t. \quad & I_t = \Upsilon_t Y_t^I \end{aligned} \tag{P5}$$

The optimization analysis in this sector draws out the common result according to which the relative price of investment equates the inverse of the investment-specific technology progress.

The capital good sector

Investment goods I_t are purchased by the perfectly competitive capital goods producers, which transform them into installed capital i_t^k, that is further sold to households at the unit price P_t^k. Firms maximize the expected discounted value of future profits subject to the technology for producing new capital

$$\begin{aligned} \max_{I_t, i_t^k} \quad & E_t \sum_{t=0}^{\infty} \beta^s \Lambda_{t+s} \left[P_{t+s}^k i_{t+s}^k - P_{t+s}^I I_{t+s} \right] \\ s.t. \quad & i_{t+s}^k = \mu_{t+s} \left[1 - S\left(\frac{I_{t+s}}{I_{t+s-1}} \right) \right] I_{t+s} \end{aligned} \tag{P6}$$

where μ_t represents the marginal efficiency of investment shock (MEI) and follows an AR(1) exogenous stochastic process

$$\log \mu_t = \rho_\mu \log \mu_{t-1} + \varepsilon_{\mu,t}, \qquad \varepsilon_{\mu,t} \sim i.i.d.N(0, \sigma_\mu^2) \tag{2.8}$$

The transformation process of the investment goods into installed capital undergoes the investment adjustment costs $S(\cdot)$. In steady state the following conditions are assumed to hold: $S = S' = 0$ and $S'' > 0$.

The employment agencies sector

Perfectly competitive employment agencies purchase specialized labor $L_t(j)$ from households at the specific wage level $W_t(j)$, and transform it into homogeneous labor, L_t, which is then sold to the intermediate-goods producers at the aggregate wage level, W_t. The employment agencies maximize their profits subject to the production function of homogeneous labor, that is

$$\begin{aligned} \max_{L_t(j)} \quad & W_t L_t - \int_0^1 W_t(j) L_t(j) dj \\ s.t. \quad & L_t = \left[\int_0^1 L_t(j)^{\frac{1}{1+\lambda_{w,t}}} dj \right]^{1+\lambda_{w,t}} \end{aligned} \tag{P7}$$

where $\lambda_{w,t}$ represents the mark-up of the wage over the marginal rate of substitution of households, and follows an AR(1) exogenous stochastic process

$$\log \lambda_{w,t} = (1-\rho_w)\log \lambda_w + \rho_w \log \lambda_{w,t-1} + \varepsilon_{w,t} - \theta_w \varepsilon_{w,t-1}, \qquad \varepsilon_{w,t} \sim i.i.d.N(0,\sigma_w^2) \tag{2.9}$$

Households

The economy is inhabited by a continuum of infinitely living households. A representative household maximizes the present value of the expected stream of logarithmic utility function with respect to current consumption, C_t, holdings of government bonds, B_t, capital utilization rate, u_t, physical capital stock, $\bar{K}_t$, and the efficiency units of maintenance, M_t, subject to the aggregate budget constraint, the law of motion of capital stock, the gross capital depreciation rate function and to the function describing the expenses in the efficiency maintenance units as follows, respectively

$$\begin{aligned}
\max_{C_t,B_t,\bar{K}_t,u_t,M_t} \quad & E_t \sum_{t=0}^{\infty} \beta^s b_{t+s} \left[\log\left(C_{t+s} - hC_{t+s-1}\right) - \varphi \frac{L_{t+s}(j)^{1+\nu}}{1+\nu} \right] \\
s.t. \quad & P_t C_t + P_t^k i_t^k + P_t^m M_t + T_t + B_t = \\
& = R_{t-1}B_{t-1} + Q_t(j) + \Pi_t + W_t(j)L_t(j) + R_t^k u_t \bar{K}_{t-1} - \frac{P_t}{\Upsilon_t} a(u_t)\bar{K}_{t-1} \\
& \bar{K}_t = (1 - \Upsilon_{t-1}^{-\sigma} D_t)\bar{K}_{t-1} + i_t^k \\
& D_t = \zeta u_t^{\eta} \left(\frac{M_t}{\bar{K}_{t-1}} \right)^{-\sigma} + \Upsilon_t^{\sigma} \bar{\delta} \\
& M_t = \tau \Upsilon_t^{-1} u_t \bar{K}_{t-1} + A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}} \bar{M}
\end{aligned} \tag{P8}$$

where h is the degree of habit formation, φ is the share parameter of labor in the utility function, ν is the inverse Frisch elasticity, T_t are lump-sum taxes, R_t is the gross nominal interest rate, $Q_t(j)$ is the net cash flow of state contingent securities, which ensures that in equilibrium consumption and the asset holdings are the same across the households, and Π_t is the per-capita profit accruing from the household's ownership of a firm. Moreover, b_t represents the intertemporal preference shock which follows an AR(1) exogenous stochastic process according to

$$\log b_t = \rho_b \log b_{t-1} + \varepsilon_{b,t}, \qquad \varepsilon_{b,t} \sim i.i.d.N(0,\sigma_b^2) \tag{2.10}$$

The rate at which capital is utilized determines the amount of effective capital which is rented by households at rate R_t^k to the firms of the intermediate-good producing sector, i.e. $K_t = u_t \bar{K}_{t-1}$. The adjustment costs of capital utilization, $a(u_t)$, following Justiniano et al. (2011), are evaluated at the Canadian dollar cost per unit of physical capital and scaled by the investment-specific technology progress in order to ensure the existence of the balanced growth path, that is $P_t a(u_t)/\Upsilon_t$. It is assumed that, in steady state, $u = 1$, $a(1) = 0$, and $\frac{a''(1)}{a'(1)} = \chi$.

I assume that the gross rate of capital depreciation, D_t, is endogenously determined by capital utilization rate, u_t, and by maintenance to capital ratio. A more intensive use of capital leads to a faster depreciation of capital, on the contrary, a higher amount of maintenance activity reduces it. Parameters η and σ represent the sensitivity of depreciation rate with respect to utilization rate and maintenance to capital ratio, respectively. I assume as well the presence of a fixed cost of depreciation, $\bar{\delta}$, i.e. the natural rate of depreciation, which is multiplied by the investment specific technology progress, Υ_t^{σ}, in oder to ensure the existence of a balanced growth path. The value of $\bar{\delta}$ is constant over time, however, its overall impact on the total gross depreciation rate is higher when an IST progress occurs. The first term on the right-hand side of the gross capital depreciation function,D_t, captures the effects of the use-related depreciation rate. The second one can be thought of as the obsolescence effect that is, when new investment goods are available on the market more new capital goods are produced, this induces the already installed capital stock to depreciate on impact. Given the main assumptions about the depreciation rate functional form, the following conditions must be satisfied: $\delta_u > 0$, $\delta_{uu} > 0$, $\delta_m < 0$, and $\delta_{mm} > 0$, which are in line with the assumptions grounded in the related literature[11]. Accordingly, the parameters of the depreciation rate function must satisfy the following assumptions: $\eta > 1$, $\sigma > 0$, $\bar{\delta} > 0$ and $\zeta > 0$. Moreover, I assume that depreciation rate is more sensitive to changes in maintenance expenses than to changes in capital utilization rate, i.e. $\sigma > \eta$. This assumption derives from the optimality conditions of my model and is supported by the estimation results obtained in Albonico et al. (2014). Finally, it has been shown that the cross-derivative of the capital depreciation function with respect to maintenance and utilization must be negative, i.e. $\delta_{um} < 0$.[12] This property is further supported by the maintenance model given that both maintenance and utilization are control variables. In fact, when a representative household decides to increase the rate of capital utilization, the rate of depreciation will also increase (given that $\delta_u > 0$). The optimal behavior of a representative household will lead to an increase in the amount of maintenance, too, in order to reduce the depreciation rate, as far as $\delta_m < 0$, and vice versa. Therefore, at optimum, maintenance and utilization must move in the same direction and, since the two have an opposite effect on depreciation, this implies that it must hold $\delta_{um} < 0$.

The law of motion of capital, differently from Justiniano et al. (2011), includes the time dependent gross depreciation rate of capital, which follows the adjustments of the IST progress. According to the optimal dynamics of the maintenance model the gross capital depreciation grows at a rate given by $\sigma\Upsilon_t$, which equates the inverse of the relative price of investment with respect to consumption weighted by the sensibility of depreciation with respect to maintenance. When a positive IST shock occurs both the price of new investment and depreciation decline on impact. This reduces the existing amount of effective capital. The higher the sensibility of depreciation rate with respect to maintenance expenses the higher the volatility of depreciation growth rate.

The last function in the households optimization problem defines the maintenance expenses

[11] See, among others, Licandro et al. (2001), and Boucekkine et al. (2009).

[12] For the analytical derivation of the sign of the depreciation rate cross-derivative see Boucekkine and Ruiz-Tamarit (2003), and for estimation highlights consult Albonico et al. (2014).

path. I assume there exists a positive relationship between efficient units of maintenance, M_t, and effective capital, $K_t = u_t\bar{K}_{t-1}$, imposing the positivity restriction on the marginal propensity to maintain, τ, i.e. $\tau \geq 0$. This implies that, when capital is used more intensively in the production process a higher amount of maintenance is required. At the same time, maintenance increases with the aging of capital stock.[13] Effective capital is multiplied by the IST progress in order to ensure the existence of a balanced growth path, and the same is done for the fixed costs of maintenance, $\bar{M}$, which are multiplied by $A_t\Upsilon_t^{\alpha/(1-\alpha)}$. I assume that there exists a strictly positive fixed cost of maintenance, $\bar{M} > 0$, in order to guarantee existence and uniqueness of the solution to the model.[14] The maintenance expenditure function satisfies the following assumptions: $M_u > 0$, $M_{\bar{K}} > 0$, and $M(u \to 0, \bar{K} \to 0) \cong \bar{M}$. When capital utilization rate tends to zero, maintenance costs approach their minimum level, $\bar{M}$, and the gross depreciation rate, D_t, tends to the level of natural depreciation rate, $\Upsilon_t^\sigma\bar{\delta}$. When the stock of capital is fully utilized (u_t tends to unity) then, for $\tau > 0$, maintenance expenses will tend to their maximum level $M_t = \tau\Upsilon_t^{-1}\bar{K}_{t-1} + A_t\Upsilon_t^{\frac{\alpha}{1-\alpha}}\bar{M}$ and depreciation will be given by $D_t = \zeta\left(\tau\Upsilon_t^{-1} + A_t\Upsilon_t^{\frac{\alpha}{1-\alpha}}\bar{M}/\bar{K}_{t-1}\right)^{-\sigma} + \Upsilon_t^\sigma\bar{\delta}$. When, instead, capital is fully utilized and households decide to keep maintenance expenses at their minimum level ($\tau = 0$ and $M_t = A_t\Upsilon_t^{\frac{\alpha}{1-\alpha}}\bar{M}$) the depreciation rate of capital is given by $D_t = \zeta\left(A_t\Upsilon_t^{\frac{\alpha}{1-\alpha}}\bar{M}/\bar{K}_{t-1}\right)^{-\sigma} + \Upsilon_t^\sigma\bar{\delta}$. It can be clearly seen that $\left(A_t\Upsilon_t^{\frac{\alpha}{1-\alpha}}\bar{M}/\bar{K}_{t-1}\right)^{-\sigma} > \left(\tau\Upsilon_t^{-1} + A_t\Upsilon_t^{\frac{\alpha}{1-\alpha}}\bar{M}/\bar{K}_{t-1}\right)^{-\sigma}$. Therefore, in the latter case a representative household will tackle with a relatively higher rate of depreciation with respect to the first case. Finally, in steady state the marginal propensity to maintain, τ, depends positively on η and negatively on σ. Therefore, when a marginal increase in maintenance induces, at margin, a relatively higher decline in depreciation the households' marginal propensity to maintenance is relatively lower. On the contrary, when a marginal increase in capital utilization brings about a high marginal acceleration in depreciation, then the steady state value of τ is relatively high. For $\eta \to 0$, $\sigma \to 0$ and $\tau \to 0$ the maintenance model reduces to the model of Justiniano et al. (2011).

Following Justiniano et al. (2011) I assume that, each household is a monopolistic supplier of a specialized labor, $L_t(j)$. Similarly to the price decision setting in the intermediate-goods sector, every period, a fraction ξ_w of households sets the wage level according to the following indexation rule

$$W_t(j) = W_{t-1}(j)\left(\pi_{t-1}e^{z_{t-1}+\frac{\alpha}{1-\alpha}v_t}\right)^{\iota_w}\left(\pi e^{\gamma_z+\frac{\alpha}{1-\alpha}\gamma_v}\right)^{1-\iota_w} \tag{2.11}$$

[13] For what it concerns the capital utilization rate, this assumption is in line with some macroeconomic studies, such as, for example, Licandro et al. (2001), Boucekkine et al. (2009) and Boucekkine et al. (2010). With regard to the stock of old installed capital, $\bar{K}_{t-1}$, instead, what is assumed is enforced, among others, by the microeconomic evidence brought out by Bitros and Flytzanis (2004) and by Bitros (2016). The two works, in fact, show analytically and empirically, respectively, that maintenance expenses depend positively on the amount of scrapped capital.

[14] Such costs may capture, for example, those intrinsic maintenance activities accomplished by the households necessary for the physical capital assets to be usable in the production process.

The remaining fraction of households, $1-\xi_w$, optimally chooses the wage level, $\tilde{W}_t(j)$, by maximizing the following present discounted value of future earnings subject to the optimal labor demand

$$\begin{aligned} &\max_{\tilde{W}_t(j)} \quad E_t \sum_{t=0}^{\infty} \beta^s \xi_w^s \left[\Lambda_{t+s} \tilde{W}_t(j) L_{t+s}(j) \pi_{t,t+s}^w - b_{t+s} \varphi \frac{L_{t+s}(j)^{1+\nu}}{1+\nu} \right] \\ &s.t. \quad L_{t+s}(j) = \left[\frac{\tilde{W}_t(j)}{W_{t+s}} \pi_{t,t+s}^w \right]^{-\frac{1+\lambda_{w,t+s}}{\lambda_{w,t+s}}} L_{t+s} \end{aligned} \tag{P9}$$

where $\pi_{t,t+s}^w = \prod_{k=0}^{s} \left(\pi_{t+k-1} e^{z_{t-1} + \frac{\alpha}{1-\alpha} \upsilon_{t-1}} \right)^{\iota_w} \left(\pi e^{\gamma_z + \frac{\alpha}{1-\alpha} \gamma_\upsilon} \right)^{1-\iota_w}$.

Public Sector

According to the Ricardian fiscal policy, the public sector finances its budget deficit through short-term bonds releases. Government expenditures are assumed to be a fraction of GDP and are given exogenously by

$$G_t = \left(1 - \frac{1}{g_t}\right) Y_t \tag{2.12}$$

where g_t is an exogenous stochastic process for government spendings

$$\log g_t = (1-\rho_g) \log g + \rho_g \log g_{t-1} + \varepsilon_{g,t}, \qquad \varepsilon_{g,t} \sim i.i.d.N(0, \sigma_g^2) \tag{2.13}$$

Monetary policy authority

The monetary authority choses the level of the nominal interest rate according to the following interest rate rule

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\rho_R} \left[\left(\frac{\pi_t}{\pi}\right)^{\phi_\pi} \left(\frac{X_t}{X_t^*}\right)^{\phi_X} \right]^{1-\rho_R} \left[\frac{X_t / X_{t-1}}{X_t^* / X_{t-1}^*} \right]^{\phi_{dX}} \varepsilon_{mp,t}, \qquad \varepsilon_{mp,t} \sim i.i.d.N(0, \sigma_{mp}^2) \tag{2.14}$$

where R is the steady state value of the nominal interest rate, X_t/X_t^* is the level of the GDP gap and $(X_t/X_{t-1})(X_t^*/X_{t-1}^*)$ is its growth rate, and $\varepsilon_{mp,t}$ represents a monetary policy shock. According to this rule the nominal interest rate responds to the deviations of inflation from its steady state level, to the level of the GDP gap and to its growth rate.

As far as the levels of labor-augmenting technology and the investment-specific technology progresses have a unit root, the main macroeconomic variables of the model, that are output, consumption, investment, maintenance, capital and real wages, fluctuate around a stochastic balanced growth path. The steady state growth rate is a linear combination of the composite technology progress $A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}}$, that is

$$\gamma^{*} = \gamma_{z} + \frac{\alpha}{1-\alpha}\gamma_{v}$$

The stationary equilibrium model is achieved de-trending the variables of the model by the composite growth trend. The non-stochastic steady state is then computed and the linear system of rational expectations equations is solved through a log-linear approximation of the model around the non-stochastic steady state.[15]

The model is composed of 20 endogenous variables in the sticky price-wage economy and of 19 endogenous variables, denoted by a 'star', in the flexible price-wage economy with null mark-up shocks.

$$\begin{bmatrix} \hat{y}_t & \hat{\bar{k}}_t & \hat{k}_t & \hat{c}_t & \hat{i}_t & \hat{\tilde{m}}_t & \hat{L}_t & \hat{\delta}_t & \hat{\rho}_t & \hat{w}_t & \hat{\tilde{w}}_t & \hat{s}_t & \hat{R}_t & \hat{\lambda}_t & \hat{\varsigma}_t & \hat{u}_t & \hat{\phi}_t & \hat{x}_t & \hat{g}_{w,t} & \hat{\pi}_t \\ \hat{y}^{\star}_t & \hat{\bar{k}}^{\star}_t & \hat{k}^{\star}_t & \hat{c}^{\star}_t & \hat{i}^{\star}_t & \hat{\tilde{m}}^{\star}_t & \hat{L}^{\star}_t & \hat{\delta}^{\star}_t & \hat{\rho}^{\star}_t & \hat{w}^{\star}_t & \hat{\tilde{w}}^{\star}_t & \hat{s}^{\star}_t & \hat{R}^{\star}_t & \hat{\lambda}^{\star}_t & \hat{\varsigma}^{\star}_t & \hat{u}^{\star}_t & \hat{\phi}^{\star}_t & \hat{x}^{\star}_t & \hat{g}^{\star}_{w,t} & \end{bmatrix}$$

The resulting optimal behaviors of the main real variables and the main steady state relations are affected by the presence of endogenous depreciation and maintenance expenses as well as by the new deep parameters. The optimal rental price of capital, fore example, depends positively on the rate of depreciation. This suggests that when depreciation accelerates the respective capital asset is more likely to be discarded sooner. The agents are prompted, therefore, to increase capital rental price in order to recover the costs deriving from higher depreciation.[16] Conversely, the shadow price of maintenance influences negatively optimal marginal product of capital. In fact, when the former increases maintenance expenses decline implying that less capital stock is repaired which, in turn, makes it less worthy. Differences from the baseline model occur also in the optimal path of the capital shadow price, which declines when agents expect depreciation to accelerate. Moreover, agents anticipate the expected rise in the relative cost of maintenance by increasing maintenance expenditures today which, in turn, lowers capital current shadow price. Finally, in the maintenance model, due to endogenous depreciation, it arises a direct (positive) impact of expected utilization on the shadow value of capital. As to the new endogenous variables, optimal depreciation rate, for example, accelerates either when more old capital stock is available on the market or when it is used more intensively in the production process. It declines, instead, when more maintenance and repair activities are undertaken by the households. Finally, optimal demand for maintenance is inversely related to its relative shadow price. On the contrary, higher depreciation and old capital both lead households to demand more maintenance, which is boosted also by the increase in the relative price of new investment.

[15] Please refer to Appendixes C, D, and E for the complete illustration of the procedure.

[16] This is in line with Jorgenson and Griliches (1967) who suggest that assets that exhibit a faster depreciation should be rented at higher prices in order to recover the costs of depreciation.

3 Estimation and results

In order to perform estimation analysis of both the baseline and maintenance models I follow the Bayesian approach with Random Walk Metropolis Algorithm. For these purposes I use the database of the Canadian economy (CANSIM) which publishes annual series for maintenance expenditures in the "Capital and repair expenditures" survey. The data used in the estimations are expressed in quarters and cover the period over 1981Q2-2015Q1.[17] In order to analyze the differences between the two models and to verify the contribution of the novelties introduced in the maintenance model I compute the impulse response functions of the main macroeconomic variables.

3.1 Data and Priors

The following eight variables are used as observables for the estimation purposes

$$\left[\Delta \log X_t \quad \Delta \log C_t \quad \Delta \log \tilde{I}_t \quad \log L_t \quad \Delta \log \frac{W_t}{P_t} \quad \pi_t \quad R_t \quad \Delta \log \frac{P_t^I}{P_t}\right]$$

where $\Delta \log X_t$, $\Delta \log C_t$ and $\Delta \log \tilde{I}_t$ are differences in logarithms of real GDP, real consumption and real investment, respectively. The latter two are defined as ratios between the corresponding nominal series and the series of the implicit price indexes for consumption of non durables, semi-durables and services and are all expressed in per-capita terms. As usual, nominal consumption defines the expenditures on non durables, semi-durables and services, while nominal investment defines the expenditures on durables and gross private domestic investment. Per-capita real GDP is expressed in chained 2007 dollars. The observable $\log L_t$ is the logarithm of per-capita hours worked, while $\Delta \log \frac{W_t}{P_t}$ is the difference in logarithms of per-capita real wages, both in the non-farm business sector. Inflation, π_t, is the quarterly difference in logarithms of consumption deflator and the nominal interest rate, R_t, is the three-months treasury bill rate. Finally, the difference in logarithms of the relative price of investment, $\Delta \log \frac{P_t^I}{P_t}$, is given by the ratio of the deflators for investment and consumption. The series for the price of investment is the average of the implicit price index on durables and gross private domestic investment. All the series are taken from the CANSIM Statistics of Canada with exception of the nominal interest rate which is from the statistics of Bank of Canada.

Note that, differently from Justiniano et al. (2011) I do not consider any structural break in the models for two reasons. First of all, I use the data since 1981 as far as several methodological changes have occurred in the CANSIM database with regard to both computation methods and data gathering. Justiniano et al. (2011), instead, cover a period over 1954Q3-2009Q1 and the structural break for the U.S. is set in 1982, which is the year when the path of the relative price of investment changes its slope. Therefore, given the affinity of the two economies and as

[17] For a detailed description of the construction of the dataset please refer to Appendix F.

it emerges observing the actual data, it is plausible to suppose that my dataset starts after the relevant changes in the path of the relative prices of investment have occurred.

I set the priors of the common parameters in the two models following Justiniano et al. (2011), with the exception of the steady state of hours worked[18]. All the parameters characterizing the persistences of the shock processes, including the parameters of the moving averages in the ARMA processes, are described by a Beta distribution. The standard errors of the innovations are described by an Inverse-gamma distribution. Prior mean of the capital share is set to 0.30 that is a value broadly used in the related literature. I set the prior means of the steady state composite growth rate, $\gamma^{\star}$, and the steady state IST growth rate, γ^{v}, respectively to 0.30 and 0.60, which correspond to the respective values after the structural break period in Justiniano et al. (2011). Both the priors follow the Normal distribution. The prior means of the price and wage stickiness, ξ_p and ξ_w, are both set such that only one third of the intermediate firms and of households can set their optimal price and optimal wage, respectively. I set the prior mean of the steady state of hours worked, $\log L^{ss}$, to 0.30 contrary to 0.00 of Justiniano et al. (2011), while keep unchanged the value of the prior standard deviation and the prior distribution form. This choice is driven by the fact that the value for the hours worked in the actual data of my dataset is on average positive. On the contrary, In Justiniano et al. (2011) the respective average value is negative over the first sub-sample, i.e. before the structural break, and positive thereafter. The prior mean of 2.00 set for the inverse Frisch elasticity is relatively high however broadly into the ranges found in the literature.

With respect to the baseline model I estimate five more parameters in the maintenance model for which I assume very dispersive priors. The sensitivity parameters of the depreciation rate function, η and σ, and the elasticity of the maintenance adjustment costs, f'', are all assumed to follow a Gamma distribution. The prior means of the sensitivity with respect to utilization, η, and with respect to maintenance, σ, are respectively 9.00 and 10.00 and the respective prior standard deviations are set to 7.00 and 10.00. The prior support of the elasticity of maintenance adjustment costs, f'', is broader with respect to the one of the investment adjustment costs, S'', with prior mean equal to 3.00 and prior standard deviation equal to 2.00, against 4.00 and 1.00, respectively. In fact, they reflect my assumption according to which a marginal change in the respective relative inputs induces more variation in the maintenance adjustment cost than in investment adjustment cost. The smooth parameter of the MST progress, ρ_d, follows a Beta distribution with prior mean 0.6 and prior standard deviation 0.2. The standard deviation of the innovation of the MST progress is described by an Inverse-gamma distribution with prior mean and prior standard deviation of 0.1 and 1, respectively. The prior supports set for the remaining parameters are, in general, in line with the related literature.

The parameters of the marginal propensity to maintain, τ, and of the depreciation rate function, ζ, are internally determined by the steady state relations. Moreover, in order to carry out a coherent comparison of the two models I set a restriction on the natural rate of

[18] All the priors are displayed in the Appendix in Table 5

depreciation, that is $\bar{\delta} = 0.025 - \zeta$. Thus, when all the new parameters of the maintenance model tend to zero ζ tends to zero, too, and the depreciation rate becomes constant and equal to 0.025 as it is in the baseline model. However, given the flexible structure of the maintenance model, such a restriction condition can be relaxed and both the values of the steady state depreciation rate and of the natural rate of depreciation can be determined by the steady state driven relations and by the estimated parameters. Otherwise, the arbitrary value of 0.025 may be set following, for example, the planned obsolescence literature. In fact, one could be interested in to assume that the capital stock approaches at most 30 years of the useful life, which would imply an average quarterly natural rate of depreciation, $\bar{\delta}$, of about 0.0083. In the present case, assuming that $\bar{\delta}$ at most equals 0.025 implies that the average useful life of capital is around 10 years when it depreciates according to the natural rate of depreciation.

Finally, I calibrate the value of the steady state government spending to GDP ratio, i.e. $(1 - 1/g)$, to 0.25 in order to match the average value of G_t/Y_t in the actual Canadian data. As to the calibration of the ratio of fixed maintenance costs to installed capital I assume an arbitrary value of 0.01. This means that households spend in fixed maintenance on average 1% of the value of current capital stock.

3.2 Posterior estimation results

The estimated modes with the respective estimated standard deviations and five degree acceptance ranges of the structural parameters and standard deviations of the innovations are listed in Table 6 in the Appendix.

Overall, the posterior estimation results confirm main assumptions set up in the maintenance model. In particular, I have assumed that the elasticity of the depreciation rate function with respect to utilization, η, is greater than unity and the posterior estimated mode of η is 1.327. Additionally, I assumed that this elasticity must be lower than the elasticity of depreciation with respect to maintenance, σ, which is as well satisfied, being the estimated mode of σ equal to 5.116. The estimated adjustment cost of maintenance, f'', is relatively lower (1.592) than the adjustment cost of investment, therefore it results to be more costly to adjust maintenance to exogenous shocks than investment (that is $1/f''$ with respect to $1/S''$). The estimated standard deviation of the maintenance specific technology progress, σ_d, is 0.247.

With respect to the baseline model, the composite steady state growth trend, $\gamma^\star$, in the maintenance model is slightly higher, 0.318 against 0.310, respectively. On the contrary, in the maintenance model the steady state growth rate of IST progress, γ^v, is slightly lower, being 0.573 against the 0.578 of the baseline model. The estimated posterior mode of capital share in the intermediate goods production function is higher in the maintenance model than in the baseline model, 0.183 and 0.141, respectively. Recall that, the composite growth trend in steady state is given by $\gamma^\star = \gamma^z + \frac{\alpha}{1-\alpha}\gamma^v$, hence, given the posterior mode for α, the contribution to the composite growth rate of the IST growth is almost double in the maintenance model with respect to the baseline model. Hours worked in steady state, $\log L^{ss}$, are 0.415 in the maintenance model and 0.163 in the baseline model. This implies that, when accounting for maintenance activity and endogenous depreciation rate, on aggregate, higher amounts of factor inputs are used in the production of intermediate-goods. The labor supply elasticities, $1/\nu$, in the maintenance and baseline models are, respectively, 0.318 and 0.257. The maintenance model result appears to be more in line with the estimates obtained in Albonico et al. (2014) which amount to 0.49 and 0.47 in the models with and without maintenance activity, respectively. The estimated elasticity of the utilization cost, χ, in the baseline model is relatively smaller, amounting to 4.819 against 5.007 in the maintenance model. Overall, these posterior estimates suggest that, the adjustment mechanism to shocks in the maintenance model, at margin, has more room of play in the sense of utilization costs and labor supply with respect to the baseline model. However, given the estimated value for the elasticity of the depreciation rate function with respect to utilization, η, that is 1.327 (1.08 in Albonico et al. (2014)), this implies that recovery through adjustments in utilization is counterproductive in terms of higher depreciation. At the same time, an increase in the utilization rate of capital raises maintenance expenditures which, in turn, downturns depreciation rate. The estimated mode for the elasticity of depreciation rate with respect to maintenance, σ, is relatively high amounting to 5.116 (whereas Albonico et al. (2014) have obtained an estimate of 19.19) suggesting that a rise in maintenance expenditures of one unit induces a marginal decrease in depreciation of about 5%. When maintenance sector is included in the model, estimated investment adjustment costs decline.[19] In fact, the estimated parameter for the adjustment cost of investment, S'', is higher in the baseline model amounting to 4.756 than the 4.136 in the maintenance model. This suggests that, in the maintenance model, the direct impact of the MEI shock in the investment Euler equation is stronger than in the baseline one.

All the other estimates are, in general, within the ranges of those found in the related literature for Canada. The estimated habit degree in consumption are 0.851 and 0.860 in the maintenance and baseline models, respectively. Both are lower than 0.94 estimated by, for example, Dorich et al. (2013)[20]. The estimated intertemporal elasticity of substitution, β, is 0.99 in both the models whereas it is 0.88 in Dorich et al. (2013). This suggests that, with respect to the model of Dorich et al. (2013), in my economy consumption is more sensitive and therefore responds faster to movements in the interest rate. Similarly to the model of Dorich et al. (2013) the maintenance model Phillips curve for consumption prices is more forward looking than backward looking with the respective coefficients for $\hat{\pi}_{t-1}$ and $\hat{\pi}_{t+1}$ of 0.22 and 0.78, given the estimated values of ι_p and β. The contributions to the price Phillips curve of real marginal cost and price mark-up disturbance, given high estimated value of price stickiness, ξ_p, are very low (0.03), which is as well in line with Dorich et al. (2013). The estimated value for the wage stickiness, ξ_w, is slightly lower in the maintenance model with respect to baseline model being, respectively, 0.652 and 0.693 (0.59 in Dorich et al. (2013)). The difference is more pronounced for the parameter of price stickiness, ξ_p, being respectively 0.823 and 0.910 (0.75 in

[19] Note that, a similar result has been achieved in Angelopoulou and Kalyvitis (2012).

[20] Dorich et al. (2013) estimate a Terms-of-Trade Economic Model for Canada with the full information estimation technique. The model is implemented by the Bank of Canada for its quarterly projection forecasts.

Dorich et al. (2013)). This suggests that, when depreciation is endogenous and the amount of maintenance can be optimally chosen a higher fraction of intermediate-goods producing firms and of households is able to optimally set the price level and wage level, respectively. On the contrary, both the estimates of the price and wage indexations, ι_p and ι_w, respectively, are higher in the maintenance model with respect to the baseline ones, which are, respectively, 0.280 against 0.267 for ι_p, and 0.100 against 0.087 for ι_w. A lower value for ι_p has been estimated in Dorich et al. (2013) which amounts to 0.06, while ι_w is 0.11. Therefore, in particular with regard to the maintenance model, it emerges that firms that do not optimize for prices and follow, instead, a general indexation rule are prompted to pay more attention to the dynamics of past price inflation relatively to the model of Dorich et al. (2013). In fact, as far as depreciation rate is time-varying, nominal interest rate, and hence inflation, will be influenced by its path impacting therefore on the dynamics of the prices. The estimated smoothing parameter of the interest rate rule is slightly lower in the maintenance model than in the baseline one, 0.853 and 0.869, respectively, and both are slightly lower than the 0.9 estimated by Dorich et al. (2013). In the maintenance model the estimated mode of the Taylor rule response to deviations of inflation, ϕ_p, is definitely higher, 2.026 against the 1.689 of the baseline model, and is broadly in line with the estimated 2.14 of Dorich et al. (2013). In the maintenance model responses to deviations of output gap level, ϕ_y, and its growth rate, ϕ_{dy} are relatively lower with respect to the baseline model (0.112 and 0.097 with respect to 0.176 and 0.134). Maintenance model estimates for Taylor rule response to output gap level is as well broadly in line with the estimated value of 0.076 in Dorich et al. (2013). The estimated smoothing parameters of the labor augmenting technology process, ρ_z, of the wage mark-up shock, ρ_w, of the intertemporal preference shock, ρ_b, and of both the moving average parameters, θ_p and θ_w, are lower in the maintenance model with respect to the baseline one. The estimated standard deviation of the MEI technology progress, σ_μ, is much larger in the maintenance model than in the baseline model, 7.873 and 4.762, respectively. The same is true for the autocorrelation parameter, ρ_μ, suggesting that in the maintenance model the MEI progress is highly volatile and persistent.

3.3 Roles of the shocks in the real business cycles

In Tables 1 and 2 are displayed the percentage contributions of the shocks to the variations of the main endogenous variables in the baseline and maintenance models, respectively.

In the baseline model the marginal efficiency of investment (MEI) technology progress is the one that mostly explains both the variations in output and investment, 21.66% and 50.44%, respectively. Moreover, it explains 17.32% of variability in hours worked, though the government spending is the most important accounting for 25.32% of hours growth. The MEI shock is negligible in the fluctuations of consumption which, again, are mainly attributable to the government spending shock (55.14%); similarly, in the fluctuations of real wages and inflation which are instead driven by the shock to the mark-up of prices (47.48% and 64.84%, respectively); and as well in the fluctuations of the nominal interest rate which are mostly explained by the monetary policy shock, amounting to 39.18%. The contribution of the investment-specific technology progress (IST) is only relevant in explaining the dynamics of the relative price of investment accounting for 100% of its variation, and its impact on the remaining main endogenous variables is lower than the impact of the labor augmenting technology progress. These findings are in line with Justiniano et al. (2011) who found that the MEI shock is responsible for 60%, 68%, and 85% of respectively output, hours and investment growths in the US economy and, similarly, explains very little of variability in consumption.

Similarly, in the maintenance model variations in output and investment are mostly due to the MEI shock with its contribution amounting to 61.24% and 68.60%, respectively. Variability in consumption and hours worked is explained by 66.82% and 27.68%, respectively. Variability in maintenance growth and depreciation is also mostly attributable to the MEI shock with 93.04% and 94.63%, respectively. As in the baseline model, fluctuations in real wages and inflation are mainly due to the price mark-up shock (55.86% and 64.25%, respectively), but the impact of the MEI shock on these variables is relatively higher in the maintenance model against the baseline model, amounting to 34.10% and 3.71% against 1.63% and 0.23%, respectively. The price mark-up shock becomes the major driver of the fluctuations in the nominal interest rate, accounting for 26.50% of its variability, while the impact of the MEI shock increases to 16.80% from 4.89% in the baseline model. Relative price of investment is entirely explained by the IST shock. The estimation results of the maintenance model confirm also that the IST progress plays an irrelevant role in driving the business cycle fluctuations, and the contribution of the labor augmenting technology progress diminished with respect to the baseline model. Finally, it is found that the maintenance-specific technology progress (MST) has a null effect with respect to all the real variables in long run, except of a relatively low impact of 0.49% on the variability of maintenance. Nevertheless, it is the fifth shock in order of importance in explaining the long run dynamics in maintenance. In short run the MST shock is the key-driver of the maintenance growth, explaining more than 40% of its variability in the first period after the shock. Its effect vanishes after more than ten periods.

These results are broadly in line with the mainstream DSGE literature which considers the investment shocks, and in general shocks to the capital accumulation process, to be the key drivers of the business cycle fluctuations. In particular, Fisher (2006), for example, states that the IST shock is responsible for most of variations in output and hours. However, as highlighted in Justiniano et al. (2011), this result is generated by the fact that he excludes the price of durable consumption from the measure of investment deflator. Therefore, by considering only the price of equipment, which is always countercyclical, the Fisher (2006) model by construction attributes more importance to the IST progress in explaining real business cycles. Indeed, in Justiniano et al. (2011) the shock that explains the most of variability in output, investment and hours is the marginal efficiency of investment technology progress. However, their model fails to generate co-movement in consumption and the same occurs in the baseline model I use for comparison purposes. Gertler and Karadi (2011) highlight the importance of the shock to the quality of capital. Their model is able to generate co-movements of output, consumption, investment and hours. Nevertheless, the qualitative behavior of the real variables in their model depends crucially on the calibration of the nominal rigidities and of the autoregressive coefficient related to the quality of capital shock process. Furlanetto and Seneca (2014) compare the roles of a shock to the quality of capital, an investment-specific technology shock and a shock to the capital depreciation rate in a DSGE model setting. They find that, the shocks to capital depreciation are the most important drivers in the macroeconomic fluctuations. However, co-movements in consumption, output, investment and hours are achieved only when capital depreciation shock is highly persistent. Therefore, in order to validate the high shock persistency, they assert that the most plausible economic interpretation for these kind of shocks would be provided by the disturbances that affect the ability of the financial intermediaries to finance the investment projects. A similar interpretation Justiniano et al. (2011) suggest for the shock to the marginal efficiency of investment. Therefore, in light of these assertions the Furlanetto and Seneca (2014) shock to the depreciation rate of capital could be related to the MEI technology shock, which affects the transformation process of the investment goods into new capital goods. In the maintenance model I presented above, the MEI shock is the one that explains the obsolescence component of capital depreciation. Finally, Greenwood et al. (1988), by adopting a standard neoclassical model enriched with a variable capital utilization rate and the depreciation-in-use hypothesis, show through a quantitative analysis that in response to a shock affecting the marginal efficiency of the newly produced capital goods only, consumption is pro-cyclical. On the contrary, as they argue, when the same type of shock hits the standard neoclassical model framework, in which capital utilization is fixed, consumption fails to co-move due to an intertemporal substitution effect away from leisure and consumption. They conclude that including variable capital utilization helps to generate co-movement in the main endogenous real variables. Nonetheless, the subsequent generation of models including the DSGEs, in which the hypothesis about variable capacity utilization is widely used, still often tackle the problem of a countercyclical response in consumption.

I conclude that a DSGE model with endogenous capital depreciation rate and control on maintenance of capital adjusts the transmission mechanism process such that an optimal behavior in consumption can be obtained in line with the observed evidence[21]. In fact, as it will be highlighted below, the DSGE maintenance model generates pro-cyclical responses to a positive MEI shock in all the main real variables and, in particular, in consumption, though the estimated autocorrelation of the MEI process is relatively higher with respect to the baseline model.

[21] Note that, the standard way to include a variable utilization rate in the macroeconomic models is by the adoption of the capital hoarding assumption. According to it, capital utilization is modeled directly in the production technology of the firms, generating thus direct positive spillovers to aggregate output when the utilization intensity increases. Howbeit, when depreciation is endogenous an additional indirect (negative) effect of the utilization rate arises. In the latter case, in fact, an increase in utilization intensity accelerates capital depreciation which destroys capital stock and, hence, lowers the aggregate output.

Tab. 1: Variance decomposition in the baseline model (percent)

	TFP	IST	MEI	Monetary policy	Government	Price mark-up	Wage mark-up	Intertemporal preference
Output	14.34	2.36	21.66	11.07	17.86	12.46	14.30	5.95
Consumption	14.82	8.19	4.26	2.06	55.14	1.36	7.24	6.93
Investment	10.09	9.46	50.44	9.41	2.08	13.75	3.57	1.20
Hours	11.10	2.32	17.32	11.09	25.32	9.22	16.19	7.45
Wages	17.54	1.56	1.63	1.56	0.14	47.48	30.00	0.080
Inflation	2.44	0.18	0.23	0.79	0.73	64.84	30.71	0.07
Interest rate	4.89	1.72	4.89	39.18	5.31	13.97	26.82	3.22
Relative price	0	100	0	0	0	0	0	0

Note: The figures represent percentage contributions of the shocks to the endogenous variables computed at the posterior mean from the unconditional variance decomposition at infinity horizon and sum up to 100 across the columns.

Tab. 2: Variance decomposition in the maintenance model (percent)

	TFP	IST	MEI	Monetary policy	Government	Price mark-up	Wage mark-up	Intertemporal preference	MST
Output	3.49	0.30	61.24	1.63	9.01	14.97	7.46	1.90	0
Consumption	3.25	1.87	66.82	0.41	19.72	3.28	3.16	1.48	0
Investment	6.88	11.49	68.60	1.27	0.04	9.34	2.28	0.11	0
Hours	4.62	6.11	27.68	2.91	20.64	19.31	14.41	4.32	0
Wages	4.52	0.72	34.10	0.15	0.06	55.86	4.57	0.02	0
Inflation	4.43	1.32	3.71	1.65	0.89	64.25	23.38	0.37	0
Interest rate	3.53	5.11	16.80	23.45	3.29	26.50	19.46	1.87	0
Relative price	0	100	0	0	0	0	0	0	0
Maintenance	1.17	2.83	93.04	0.09	0.01	2.14	0.23	0.01	0.49
Depreciation	0.97	3.14	94.63	0.06	0	1.00	0.19	0.01	0

Note: The figures represent percentage contributions of the shocks to the endogenous variables computed at the posterior mean from the unconditional variance decomposition at infinity horizon and sum up to 100 across the columns.

3.4 Optimal convergence dynamics

In order to investigate the optimal behavior of the main macroeconomic variables and assess the performance of the maintenance model against the baseline model I compute the impulse response functions to the shocks included in the models. For the calibration of parameters I use the estimated median values and the standard deviations of the shocks, normalized to one, of the respective models. Fig.2 displays the IRFs of the main macroeconomic variables to a positive marginal efficiency of investment technology shock.[22]

Overall, as it can be observed in Fig. 2, both the models deliver well hump-shaped curves. A positive shock to the MEI technology process increases on impact the amount of new capital above its steady state level. As a consequence, Tobin's q, $\hat{q}_t = \hat{\phi}_t - \hat{\lambda}_t$ and the equilibrium cost of new capital decline. Given the following equilibrium condition for the real interest rate

$$\begin{aligned} \hat{R}_t - \hat{\pi}_{t+1} = & -\hat{q}_t + \left[1 - \bar{D} - \beta e^{-\left(\gamma_z + \left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v\right)}\tau \bar{A}\right]\hat{q}_{t+1} + \frac{\rho}{\rho-\tau}\bar{D}\hat{\rho}_{t+1} + \beta e^{-\left(\gamma_z + \left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v\right)}\bar{B}\hat{u}_{t+1} + \\ & -\beta e^{-\left(\gamma_z + \left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v\right)}\left(\bar{C} + e^{\sigma\gamma_v}\sigma\bar{\delta}\right)\hat{\delta}_{t+1} - \left[1 + \beta e^{-\left(\gamma_z + \left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v\right)}\sigma\bar{C}\right]\rho_v\hat{v}_t + \\ & + \left[\beta e^{-\left(\gamma_z + \left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v\right)}\tau\bar{A} - \frac{\tau}{\rho-\tau}\bar{D}\right]\left(\hat{\varsigma}_{t+1} - \hat{\lambda}_{t+1}\right) \end{aligned} \tag{3.1}$$

the inverse relation with the Tobin's q induces real interest rate to increase on impact, which in turn attracts new investment. In the baseline model this interest rate effect is strong enough to generate a substitution effect away from consumption, so agents find it optimal to postpone consumption of final goods and to increase investment. Consequently, consumption decreases on impact of the shock drawing an increasing equilibrium path and overshoots above the steady state level after a few periods, following the recovery of real interest rate. The optimal level of labor increases on impact of the shock given a higher marginal utility of labor on the supply side, and a lower price mark-up of the intermediate goods producers, because of increased real marginal costs, on the demand side.

On the contrary, in the maintenance model current consumption increases on impact of the shock. This is explained by a relatively stronger wealth effect combined with a weaker interest rate effect. Therefore, the optimal level of investment increases on impact of the shock though less than in the baseline model. The increased economy's productive capacity delivers a slightly positive impact response in the capital utilization rate which, in turn, rises on impact both effective capital and its marginal productivity. Thus, given that real marginal costs of the intermediate goods producers immediately rise, and that the presence of nominal rigidities do not allow the firms to promptly adjust for the prices, the price mark-up declines. As a consequence, co-movement of consumption and hours in the maintenance model is ensured through the labor market equilibrium condition given the countercyclical behavior of the mark-ups. The increase in the level of new capital rises on impact the stock of installed capital which over the next periods affects effective capital. Thus, after a weak positive response on impact of the shock due to the impact increase in the utilization rate, the optimal effective capital keeps on growing and reverts back to its steady state level after several periods. Consequently, the marginal product of capital, after a weak on impact positive response, declines below its steady state level and inverts its route as the effective capital recovers slowly to its respective

[22] The IRFs to the other shocks and a brief description of the optimal dynamics of the model are displayed in Appendix G.

equilibrium level. Capital utilization rate depicts a hump-shaped path similarly to the rental rate of capital. Overall, the positive wealth effect and the relatively weaker substitution effect in the maintenance model allows for co-movements in consumption, output, investment and hours. Contrarily, in the baseline model the co-movement in investment, hours and output comes out at the cost of countercyclical consumption. Moreover, the persistent increase of the existent capital stock creates in the maintenance model a positive impact response and an increasing path in real maintenance inducing the next period capital depreciation rate to accelerate, which further pushes up the demand for maintenance. Therefore, maintenance and investment are complements in short run[23].

Endogenous depreciation and the presence of maintenance sector amplify the convergence dynamics of the equilibrium paths in response to a positive MEI shock with respect to the baseline model. Convergence for almost all the considered real variables is delayed, in part, because of a relatively higher estimated persistence in the shock with respect to the baseline model. The optimal response paths of effective capital, consumption, real wages and output are significantly amplified although the estimated nominal and real frictions in the maintenance model are relatively lower. The optimal path of investment in the maintenance model is relatively smoother due to lower estimate of the investment adjustment cost. A relatively higher estimated steady state elasticity of the utilization costs, instead, delivers relatively stronger response path for the utilization rate. On the contrary, given the relatively higher estimates of the nominal rigidities in the baseline model, agents find it optimal to increase the hours of work more than in the maintenance model.

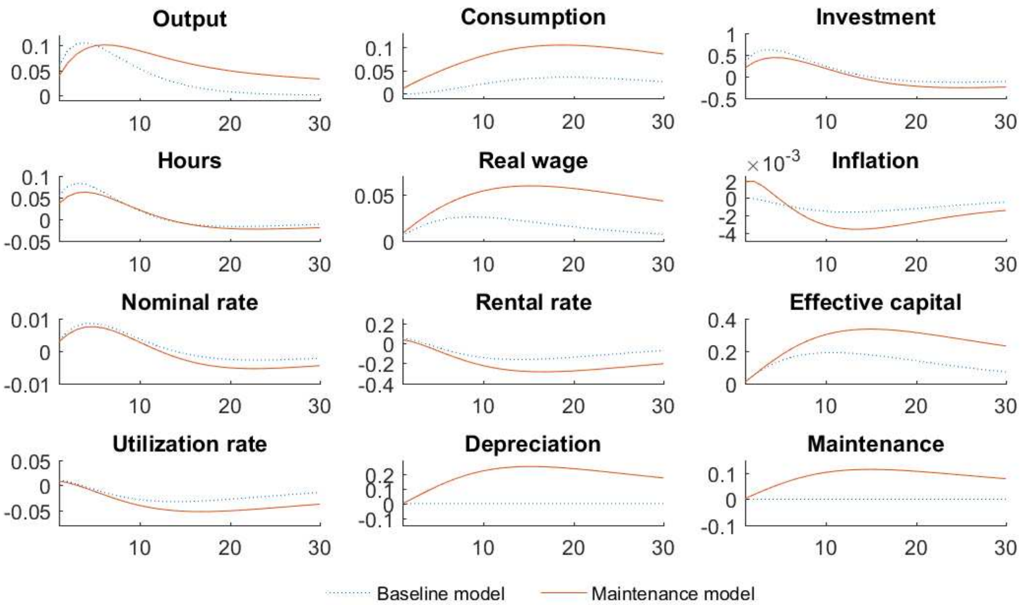
Finally, convergence towards the respective equilibrium levels of the main endogenous variables, with exception of hours, is delayed in the maintenance model. In the baseline model, for example, output reaches its equilibrium level after around 20 periods. In the maintenance model convergence is not accomplished before than 30 periods. Optimal investment in the maintenance model, after a positive response on impact of about 0.20%, falls below its steady state level after 14 periods, thereafter recovery to equilibrium is very slow. In the baseline model overshooting occurs after 15 periods and recovery is relatively faster. A similar behavior is obtained for the nominal interest rate. The impulse response function of inflation in the maintenance model is mildly cyclical as a consequence of the accelerated depreciation rate and a more volatile price mark-up.

3.5 Models fit

In the present section I compare some estimated statistics both for the maintenance and the baseline models with those generated by the actual data for the variables treated as observables and expose also the estimates for maintenance growth and the depreciation rate.

[23] Maintenance and investment behave as complements also in the case of the neutral labor augmenting technology shock, while they are gross substitutes in response to the IST shock. For details please see the Appendix.

Fig. 2: Impulse response functions to one standard deviation shock to MEI



In Table 3 are presented the posterior medians of the standard deviations in absolute values as well as those relative to output growth. In Table 4 I refer to the autocorrelation values and to the cross-correlations with output growth. In general, it can be observed that maintenance model mimics Canadian economy much better with respect to the baseline model. The estimated volatility of output growth in maintenance model perfectly matches the respective empirical moment (0.73), which instead is overpredicted by the baseline model (0.99). Consumption growth volatility is as well better fitted by the maintenance model though underpredicted (0.50) while again overpredicted by the baseline model (0.70). The latter one performs slightly better with respect to volatilities of real investment (2.33), wage growth (0.82), and inflation (0.63). On the contrary, the empirical volatilities of hours worked, nominal interest rate and the relative investment price are better fitted by the estimates in the maintenance model which are, respectively, 2.24, 0.58, and 0.66.

The estimated relative standard deviations in the maintenance model better fit the respective empirical ones for what it concerns real investment growth (3.44), real wage growth (1.19), inflation (0.82), nominal interest rate (0.78), and the relative price of investment (0.90). All the estimated relative volatilities are underpredicted by the baseline model. The maintenance model, on the contrary, overpredicts the empirical relative volatilities of real investment, hours worked, and wage growth.

In the maintenance model the estimated autocorrelations, which are displayed in Table 4, resemble better those of the actual data for output growth, hours worked and the nominal interest rate. In the case of investment relative price the estimated autocorrelation is fitted slightly better by the baseline model, 0.41, being 0.42 in the maintenance model and 0.30 in the actual data. Similarly, the baseline model matches slightly better the persistence of real wage growth, which is 0.33 against 0.35 in the maintenance model and 0.24 in the data. The empirical autocorrelation of consumption growth, 0.002, is highly overpredicted by both the models with a much higher estimated persistence in the maintenance model (0.73). The autocorrelation of hours worked is perfectly matched by the maintenance model, wheres that of inflation is perfectly matched by the baseline model.

Similarly, the overall estimated contemporaneous cross-correlations with output growth are performed better in the maintenance model. In particular, a better fit occurs in the case of investment growth which is estimated to amount to 0.74, while it is 0.61 in the data. The baseline model, on the contrary, underpredicts the cross-correlation for investment which is around 0.42. The empirical cross-correlation of hours worked (0.04) is as well better suited by the maintenance model (0.08), whereas it is highly overpredicted by the baseline model (0.11). The latter one fits much better with respect to the maintenance model the cross-correlations of real wage growth (0.24) and inflation (-0.19). On the contrary, the maintenance model perfectly matches the empirical cross-correlation of the nominal interest rate (-0.25). Finally, the baseline model totally fails to capture the cross-correlation of the relative price of investment which is estimated to be negative, -0.06, although the respective one in the maintenance model, 0.06, highly underpredicts the empirical one, 0.21.

Hence, overall, the estimated moments in the maintenance model fit the respective empirical ones much better in comparison to the baseline model. With regard to the estimates produced for the maintenance and the depreciation growth rates in the maintenance model no direct comparisons can be made with the respective empirical measures because of the lack of actual data. However, it could be inferred that the maintenance model preforms fairly well as far as it almost confirms the theoretical insights. The estimated volatility of maintenance growth rate, 0.80, is higher, in fact, with respect to both the estimated and empirical volatilities of real output growth, 0.73. Furthermore, as the relative literature suggests,[24] real maintenance growth rate is less volatile than the growth rate of real investment (2.50). My estimation results confirm as well the literature insights according to which depreciation growth rate is highly volatile, which is estimated to be around 3.75.[25] Moreover, the estimated persistency of capital depreciation resembles the one estimated for the relative price of investment which, as a consequence of the maintenance model assumptions, determines the trend of the optimal depreciation path. The growth rate of real maintenance is found to be the less persistent one among all the other endogenous variables, with an estimated autocorrelation of about 0.16. On the contrary, the contemporaneous cross-correlation of maintenance with output is estimated to be around 0.50. The contemporaneous cross-correlation between depreciation growth and output growth is almost null (0.03). This estimation result could be partly explained by the factor hoarding hypothesis. In fact, factor hoarding implies that the current capital stock,

[24] See, for example, the highlights in McGrattan and Schmitz (1999).
[25] See, for example, Albonico et al. (2014).

which is partly destroyed by current depreciation, is used in the production of the intermediate goods over the next period. This implies that the final output produced in period t is affected by the capital depreciation rate which has occurred in period $t-1$. Even though, the estimated at the median autocorrelation function between capital depreciation growth rate and output growth rate, $(dy_t d\delta_{t-k})$, delivers relatively low figures (though higher than the contemporaneous cross-correlation), averaging around 0.08, 0.07, 0.05, and 0.03 for the first, second, third, and fourth order cross-correlations, respectively. On the contrary, the estimation results confirm the existence of a high positive correlation between capital depreciation rate and the declining rate of the relative price of investment which in the median is around 0.98.[26]

Finally, given the posterior mean values of the estimated parameters, the average quarterly depreciation rate results to be 2.95 which implies an annual average depreciation of about 11.80%. It should be noticed that this value does not departure too much from the 10% assumed in Justiniano et al. (2011). This could be attributed to the restriction I have imposed on the definition of the natural rate of depreciation in order to be able to switch from one model to the other. I have furthermore computed an estimation exercise of the maintenance model estimating the parameter of the steady state depreciation rate. For this purpose I have assumed that the steady state depreciation rate follows a normal distribution with prior mean and prior standard deviation both of 0.05. In this case the estimation results do not change qualitatively though the annualized posterior mean of depreciation amounts to almost 14%.

Tab. 3: Standard deviations of actual and estimated series

	Standard deviation			Standard deviation relative to output		
	Data	Baseline model	Maintenance model	Data	Baseline model	Maintenance model
		Median	Median		Median	Median
Output growth	0.73	0.99	0.73			
Consumption growth	0.56	0.70	0.50	0.77	0.71	0.68
Investment growth	2.27	2.33	2.50	3.10	2.35	3.44
Hours	1.98	2.43	2.24	2.70	2.45	3.08
Wage growth	0.78	0.82	0.87	1.07	0.83	1.19
Inflation	0.65	0.63	0.60	0.89	0.64	0.82
Interest rate	1.09	0.54	0.58	1.49	0.54	0.78
Relative price	0.72	0.85	0.66	0.99	0.86	0.90
Maintenance growth			0.80			1.10
Depreciation growth			3.75			5.16

Note: Empirical moments are computed using Canadian data over 1981Q2-2015Q1. The moments of the models are computed at the median of 80,000 draws from estimated posterior distribution.

[26] See, among others, Boucekkine et al. (2009).

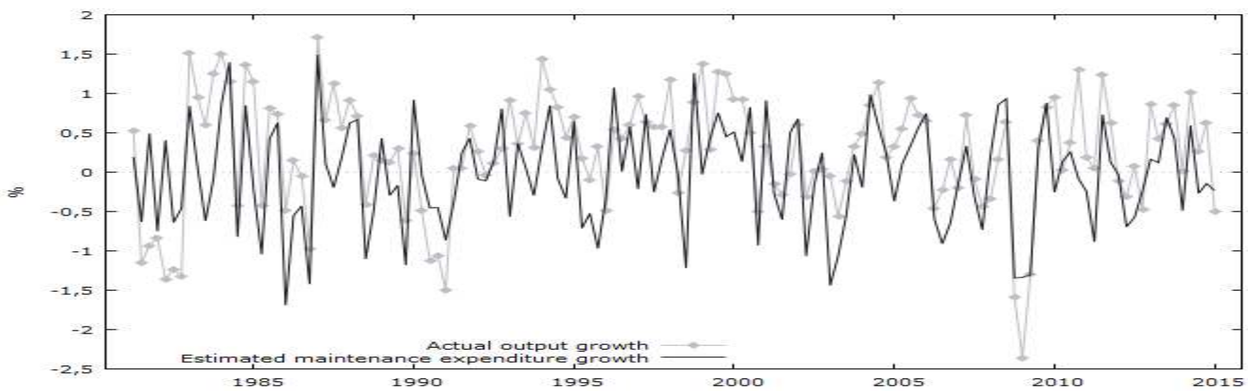
Tab. 4: Correlations of actual and estimated series

	Autocorrelation			Correlation with output growth		
	Data	Baseline model	Maintenance model	Data	Baseline model	Maintenance model
		Median	Median		Median	Median
Output growth	0.43	0.23	0.58			
Consumption growth	0.002	0.26	0.73	0.45	0.56	0.75
Investment growth	0.52	0.65	0.77	0.61	0.42	0.74
Hours	0.95	0.92	0.95	0.04	0.11	0.08
Wage growth	0.24	0.33	0.35	0.21	0.24	0.36
Inflation	0.58	0.58	0.70	-0.20	-0.19	-0.41
Interest rate	0.98	0.88	0.92	-0.25	-0.23	-0.25
Relative price	0.30	0.41	0.42	0.21	-0.06	0.06
Maintenance growth			0.16			0.50
Depreciation growth			0.42			0.03

Note: Empirical moments are computed using Canadian data over 1981Q2-2015Q1. The moments of the models are computed at the median of 80,000 draws from estimated posterior distribution.

Fig. 3 plots the estimated series for the maintenance expenditures growth rate implied by the maintenace model and the actual growth rate of Canadian real GDP. As it can be observed, most of the peaks and troughs in real output variability are captured fairly well by the movements in real maintenance.

Fig. 3: Actual real output and estimated real maintenance expenditures growth rates for Canada (1981:II-2015:I)



Source: Real output growth rate from Statistics of Canada (CANSIM). Growth rate of real maintenance expenditures from estimation results.

4 Conclusions

I build and estimate a Dynamic Stochastic General Equilibrium model assuming an endogenous depreciation rate of capital and the presence of a capital maintenance sector affected by adjustment costs to maintenance. For comparison purposes I also consider a baseline model in which I abstract from the maintenance sector and assume a constant rate of depreciation. The estimation exercises are carried out following the Bayesian approach with Metropolis-Hasting algorithm. The sampling dataset is built up using Canadian series over the period 1981Q2-2015Q1 at quarterly frequency.

The novelty introduced in the maintenance model analytical framework concerns the fact that capital depreciation rate exhibits an increasing trend path. It is the first attempt in the related literature dealing with an endogenous and non-stationary depreciation. The optimality conditions of the model deliver a growth rate for depreciation that depends on the investment-specific technology progress, weighted by the sensibility of depreciation to changes in maintenance expenditures. It is worth to stress that, over the latest decades, it has been observed a persistent decline in the relative price of investment coupled by an acceleration in capital depreciation. The maintenance model is able to capture this behavior basing on the assumption that, in a perfectly competitive market, relative price of investment is inversely related to the investment-specific technology progress.

Following the mainstream literature I define an analytical expression for depreciation in terms of maintenance and utilization. Specifically, depreciation is positively related to the capital utilization rate and inversely related to maintenance to capital ratio. One implication deriving from the explicit definition of capital depreciation is that, now, the utilization rate affects the final aggregate output through two different channels. There is one positive effect which affects directly the aggregate production process by the means of hoarding assumption. The second indirect propagation channel is expressed by the capital accumulation process. In fact, when utilization rate increases depreciation accelerates reducing the level of capital stock available on the market which, in turn, lowers aggregate output. Furthermore, agents are allowed to repair and renovate the damaged or obsolete capital stock through the maintenance activities, which slow down the rate of depreciation. Note that, maintenance goods are assumed to repair the existing capital stock but not to create new units of capital.

The estimation exercises performed on the Canadian economy show that the optimal behavior of some real endogenous variables changes both qualitatively and quantitatively in the maintenance model with respect to the baseline model with constant depreciation and no maintenance of capital. Most importantly, the maintenance model is able to generate a pro-cyclical behavior in real consumption in response to a positive technology shock that drives the business cycle. It is also found that the relationships between capital depreciation rate, maintenance and investment differ in response to the type of the shock that hits real economy. When considering the shock to marginal efficiency of investment, that is the key driver of the real business cycles explaining more than sixty percent of output growth in the long-run, real investment and maintenance behave as complements. As far as the MEI technology shock affects the qualitative innovations of new investment and, hence, of capital, it accelerates the rate of capital depreciation through the obsolescence effect. Optimal demand for maintenance increases as a result of a higher depreciation. Therefore, real investment, consumption, maintenance and depreciation co-move with real output. In general, the optimal paths are well hump-shaped and are amplified with respect to the baseline model. Convergence is delayed due to changes in the propagation mechanism of the shocks. On the contrary, in response to the investment-specific technology shock maintenance and investment act as substitutes. The rate of depreciation, real consumption and maintenance are all pro-cyclical, while investment is countercyclical. The decline in depreciation, in this case, is explained by the fact that this type of shock rises the average service life of existing capital since it affects the marginal cost of production of one extra unit of capital. This implies that a higher quantity of capital of the same quality is available on the market, which in aggregate terms means that capital stock becomes "younger". The labor-augmenting technology shock produces co-movement in all the considered real variables, so maintenance and investment are complements. The maintenance model considers as well a shock that hits the technology of production of maintenance goods, which generates a countercyclical behavior in real consumption, investment and the capital depreciation rate due to a strong intertemporal substitution effect away from real maintenance. This suggests that, in this case, maintenance and investment act as substitutes. This shock in long run has almost null effect on the growth rates of the main macroeconomic variables. However, it is the key driver of real maintenance growth in short run, explaining on average more than 25% per period of real maintenance variation over the subsequent five periods after the shock.

The estimation results show furthermore that most of the variations in the growth rates of the main real variables are explained by the marginal efficiency of investment technology progress. Specifically, the MEI shock explains more than 60% of variation in output, consumption and investment, and more than 90% of variation in maintenance and depreciation. These results are in line with Justiniano et al. (2011) except for consumption growth that, in their model, is driven by the shock to intertemporal preference. Moreover, similarly to them, I have found that the role of investment-specific technology progress is negligible.

In general, the posterior estimation analyses confirm the hints advanced in the related literature with regard to the qualitative behavior of capital depreciation. In particular, I show that there exists a positive correlation between the declining rate of the relative price of investment and the depreciation rate of capital. Depreciation rate is found to be more volatile than investment. However, the maintenance model fails to predict the strong pro-cyclicality between depreciation and output.

To conclude, the model setting implemented in this work may represent a good starting point in order to include variations in capital depreciation rate in the endogenous growth models. As far as more and more thoughts arise in favor of an increasing trend over time of depreciation, and in general of a depreciation that varies over time, this model structure provides useful instruments able to tackle with an accumulation process of capital stock with time-dependent depreciation rate. So, the model can be used for the assessment of the economic performance.

Moreover, the availability of consistent estimates of capital stock and depreciation are crucial for the capital taxation codes. In fact, as it is stressed in, for example, Doms et al. (2004) biased estimates of depreciation may result in misleading tax policy for capital income. The strategic investment decisions undertaken at the firm level are also dependent on the path of the depreciation rate, especially during technological booms. Tevlin and Whelan, 2003 claim that when an acceleration in the depreciation rate occurs firms need to invest more in order to preserve a given level of capital stock. So, I show that capturing the dynamics of capital depreciation in a Dynamic Stochastic General Equilibrium model improves efficiently the estimated equilibrium dynamics of the main real endogenous variables.

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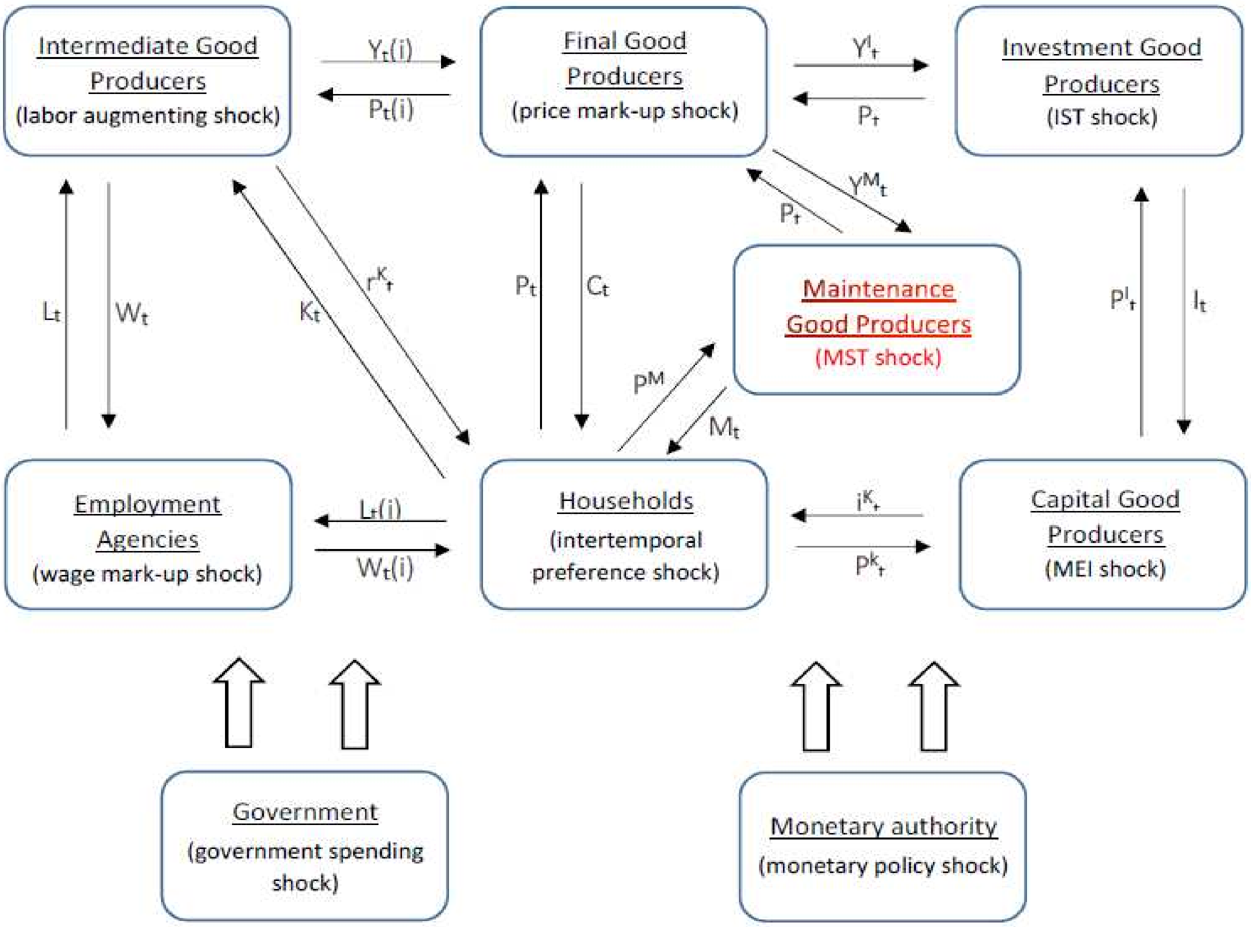
A APPENDIX: Model scheme, Priors and Posteriors



Tab. 5: Prior distributions of structural parameters and standard deviations of the shocks

Parameter	Description	Prior shape	Prior mean	Prior Std
α	Capital share	N	0.30	0.05
ι_p	Price indexation	B	0.50	0.15
ι_w	Wage indexation	B	0.50	0.15
$\gamma^\star$	SS composite technology	N	0.30	0.025
γ^v	SS IST growth rate	N	0.60	0.025
h	Consumption habit	B	0.50	0.10
λ_p	SS prices mark-up	N	0.15	0.05
λ_w	SS wages mark-up	N	0.15	0.05
$\log L^{ss}$	SS hours	N	0.30	0.50
$100(\pi-1)$	SS quarterly inflation	N	0.50	0.10
$100(\beta^{-1}-1)$	Discount factor	G	0.25	0.10
ν	Inverse Frisch elasticity	G	2.00	0.75
ξ_p	Price stickiness	B	0.66	0.10
ξ_w	Wage stickiness	B	0.66	0.10
χ	Utilization cost elasticity	G	5.00	1.00
S''	Investment adjustment costs	G	4.00	1.00
ϕ_p	Inflation Taylor rule	N	1.70	0.30
ϕ_y	Output Taylor rule	N	0.13	0.05
ϕ_{dy}	Output growth Taylor rule	N	0.125	0.05
η	Elasticity of depreciation wrt utilization	G	9.00	7.00
σ	Elasticity of depreciation wrt maintenance	G	10.00	10.00
f''	Maintenance adjustment costs	G	3.00	2.00
ρ_d	Persistence maintenance cost	B	0.60	0.20
ρ_R	Persistence Taylor rule	B	0.60	0.20
ρ_z	Persistence Neutral technology	B	0.40	0.20
ρ_g	Persistence government spending	B	0.60	0.20
ρ_v	Persistence IST	B	0.20	0.10
ρ_p	Persistence price mark-up	B	0.60	0.20
ρ_w	Persistence wage mark-up	B	0.60	0.20
ρ_b	Persistence intertemporal preference	B	0.60	0.20
ρ_μ	Persistence MEI	B	0.60	0.20
θ_p	MA price mark-up	B	0.50	0.20
θ_w	MA wage mark-up	B	0.50	0.20
σ_{mp}	Monetary policy Std	I	0.10	1.00
σ_z	Neutral technology growth Std	I	0.50	1.00
σ_g	Government spending Std	I	0.50	1.00
σ_v	IST growth Std	I	0.50	1.00
σ_μ	MEI growth Std	I	0.50	1.00
σ_p	Price mark-up Std	I	0.10	1.00
σ_w	Wage mark-up Std	I	0.10	1.00
σ_b	Intertemporal preference Std	I	0.10	1.00
σ_d	Maintenance specific growth Std	I	0.10	1.00

Tab. 6: Posterior estimates of the baseline and maintenance models

	Baseline model			Maintenance model		
Parameter	Mode	s.d.	[5, 95]	Mode	s.d.	[5, 95]
α	0.141	0.018	[0.11, 0.17]	0.183	0.020	[0.14, 0.23]
ι_p	0.267	0.078	[0.10, 0.40]	0.280	0.096	[0.12, 0.47]
ι_w	0.087	0.035	[0.04, 0.17]	0.100	0.041	[0.04, 0.19]
$\gamma^\star$	0.310	0.024	[0.26, 0.35]	0.318	0.024	[0.27, 0.35]
γ^v	0.578	0.025	[0.53, 0.63]	0.573	0.025	[0.53, 0.62]
h	0.860	0.024	[0.80, 0.90]	0.851	0.025	[0.78, 0.90]
λ_p	0.224	0.043	[0.13, 0.30]	0.227	0.043	[0.13, 0.31]
λ_w	0.147	0.049	[0.05, 0.23]	0.154	0.045	[0.06, 0.24]
$\log L^{ss}$	0.163	0.429	[-0.63, 0.99]	0.415	0.471	[-0.51, 1.22]
$100(\pi - 1)$	0.544	0.086	[0.39, 0.71]	0.529	0.085	[0.36, 0.69]
$100(\beta^{-1} - 1)$	0.413	0.096	[0.24, 0.62]	0.454	0.111	[0.26, 0.69]
ν	3.884	0.843	[2.50, 5.79]	3.144	0.922	[1.85, 5.16]
ξ_p	0.910	0.021	[0.84, 0.94]	0.823	0.058	[0.74, 0.90]
ξ_w	0.693	0.073	[0.55, 0.84]	0.652	0.073	[0.52, 0.84]
χ	4.819	0.976	[3.20, 7.07]	5.007	0.988	[3.32, 7.46]
S''	4.756	0.902	[3.40, 7.15]	4.136	0.799	[3.03, 6.30]
ϕ_p	1.689	0.239	[1.28, 2.16]	2.026	0.201	[1.55, 2.42]
ϕ_y	0.176	0.042	[0.09, 0.26]	0.112	0.061	[0.04, 0.24]
ϕ_{dy}	0.134	0.038	[0.06, 0.20]	0.097	0.040	[0.03, 0.19]
η				1.327	1.534	[0.01, 7.69]
σ				5.116	1.985	[3.19, 13.44]
f''				1.592	1.426	[0.12, 5.61]
ρ_d				0.668	0.272	[0.24, 0.97]
ρ_R	0.869	0.018	[0.83, 0.90]	0.853	0.023	[0.82, 0.89]
ρ_z	0.061	0.049	[0.005, 0.17]	0.056	0.045	[0.01, 0.15]
ρ_g	0.982	0.005	[0.97, 0.99]	0.986	0.005	[0.98, 0.99]
ρ_v	0.413	0.070	[0.28, 0.54]	0.435	0.071	[0.28, 0.55]
ρ_p	0.950	0.040	[0.65, 0.99]	0.959	0.030	[0.86, 0.99]
ρ_w	0.915	0.053	[0.70, 0.97]	0.837	0.064	[0.59, 0.92]
ρ_b	0.183	0.087	[0.05, 0.39]	0.151	0.079	[0.04, 0.37]
ρ_μ	0.699	0.086	[0.54, 0.85]	0.979	0.013	[0.93, 0.99]
θ_p	0.930	0.051	[0.46, 0.97]	0.839	0.100	[0.62, 0.94]
θ_w	0.864	0.079	[0.55, 0.95]	0.733	0.105	[0.32, 0.86]
σ_{mp}	0.250	0.017	[0.22, 0.29]	0.242	0.016	[0.22, 0.28]
σ_z	0.860	0.058	[0.75, 0.99]	0.905	0.065	[0.79, 1.04]
σ_g	0.661	0.041	[0.60, 0.76]	0.627	0.039	[0.56, 0.72]
σ_v	0.727	0.044	[0.65, 0.84]	0.723	0.044	[0.65, 0.82]
σ_μ	4.762	1.058	[3.28, 7.79]	7.873	1.144	[6.22, 11.28]
σ_p	0.346	0.028	[0.27, 0.39]	0.319	0.031	[0.25, 0.38]
σ_w	0.265	0.028	[0.21, 0.31]	0.275	0.031	[0.21, 0.33]
σ_b	0.139	0.024	[0.09, 0.18]	0.142	0.022	[0.09, 0.18]
σ_d				0.247	0.098	[0.13, 0.79]
log Posterior	-1141.65			-1139.74		

Note: Following Justiniano et al. (2011), posterior estimates are obtained from 2 chains of 80,000 draws. The initial 50,000 draws are discarded and kept one every five subsequent draws imposing the scale for the jumping distribution to 0.31 for both the models.

B APPENDIX: Analytical structure of the model and optimality conditions

In the present section of the appendix I analytically derive all the first order conditions of the model.

Final good sector
Recalling the problem setting (P1) the Lagrangian is as follows

$$\mathcal{L} = P_t Y_t - \int_0^1 P_t(i) Y_t(i) di + \Lambda_t \left\{ \left[\int_0^1 Y_t(i)^{\frac{1}{1+\lambda_{p,t}}} di \right]^{1+\lambda_{p,t}} - Y_t \right\} \tag{B.1}$$

The first order condition (FOC) with respect to Y_t is, therefore

$$\frac{\partial \mathcal{L}}{\partial Y_t} = P_t - \Lambda_t$$

$$P_t - \Lambda_t = 0$$

$$P_t = \Lambda_t \tag{B.2}$$

where the latter expression simply equates the consumption price to its shadow value.
The FOC with respect to $Y_t(i)$, using (B.2), is as follows

$$\frac{\partial \mathcal{L}}{\partial Y_t(i)} = -P_t(i) + \Lambda_t \left[\int_0^1 Y_t(i)^{\frac{1}{1+\lambda_{p,t}}} di \right]^{1+\lambda_{p,t}-1} Y_t(i)^{\frac{1}{1+\lambda_{p,t}}-1}$$

$$P_t(i) = P_t Y_t(i)^{-\frac{\lambda_{p,t}}{1+\lambda_{p,t}}} \left\{ \left[\int_0^1 Y_t(i)^{\frac{1}{1+\lambda_{p,t}}} di \right]^{1+\lambda_{p,t}} \right\}^{\frac{\lambda_{p,t}}{1+\lambda_{p,t}}}$$

$$Y_t(i) = \left[\frac{P_t(i)}{P_t} \right]^{-\frac{1+\lambda_{p,t}}{\lambda_{p,t}}} Y_t \tag{B.3}$$

Expression (B.3) defines the optimal intermediate good demand function, which negatively relates it to the relative price of consumption.
Starting with the zero profit condition and substituting for equation (B.3), the following passages derive the optimal final good aggregate price, P_t,

$$P_t Y_t - \int_0^1 P_t(i) \left[\frac{P_t(i)}{P_t} \right]^{-\frac{1+\lambda_{p,t}}{\lambda_{p,t}}} Y_t di = 0$$

$$P_t - P_t^{\frac{1+\lambda_{p,t}}{\lambda_{p,t}}} \int_0^1 P_t(i) P_t(i)^{-\frac{1+\lambda_{p,t}}{\lambda_{p,t}}} di$$

$$P_t = \left[\int_0^1 P_t(i)^{-\frac{1}{\lambda_{p,t}}} di \right]^{-\lambda_{p,t}} \tag{B.4}$$

Intermediate good sector

Recalling problem setting P2 from the main text, the Lagrangian is written as follows

$$\mathcal{L} = P_t(i)Y_t(i) - W_tL_t(i) - R_t^k K_t(i) + MC_t(i)\left[A_t^{1-\alpha}K_t(i)^{\alpha}L_t(i)^{1-\alpha} - A_t\Upsilon_t^{\frac{\alpha}{1-\alpha}}F - Y_t(i)\right] \tag{B.5}$$

The FOCs with respect to labor, $L_t(i)$, and effective capital $K_t(i)$, are, respectively

$$\frac{\partial \mathcal{L}}{\partial L_t(i)} = -W_t + MC_t(i)\left(1-\alpha\right)A_t^{1-\alpha}K_t(i)^{\alpha}L_t(i)^{-\alpha} = 0$$

$$MC_t(i) = \frac{W_t}{1-\alpha}A_t^{-(1-\alpha)}\left[\frac{K_t(i)}{L_t(i)}\right]^{-\alpha} \tag{B.6}$$

$$\frac{\partial \mathcal{L}}{\partial K_t(i)} = -R_t^k + MC_t(i)\alpha A_t^{1-\alpha}K_t(i)^{\alpha-1}L_t(i)^{1-\alpha} = 0$$

$$MC_t(i) = \frac{R_t^k}{\alpha}A_t^{-(1-\alpha)}\left[\frac{K_t(i)}{L_t(i)}\right]^{1-\alpha} \tag{B.7}$$

where the Lagrangian shadow value, $MC_t(i)$, represents the nominal marginal cost, which is common to all the intermediate sector firms, i.e. $MC_t(i) = MC_t$. The Cobb-Douglas production technology, in fact, satisfies the desirable property of duality which, furthermore, implies that the average variable cost equals the nominal marginal cost. Consequently, capital to labor ratio is the same among all the intermediate good producers, and it is obtained by combining (B.7) with (B.6), so that

$$\frac{K_t}{L_t} = \frac{W_t}{R_t^k}\frac{\alpha}{1-\alpha} \tag{B.8}$$

Then, substituting back (B.8) into either of the two FOCs for capital and labor I obtain the following expression for nominal marginal cost, MC_t,

$$MC_t = \frac{W_t}{1-\alpha}A_t^{-(1-\alpha)}\left[\frac{W_t}{R_t^k}\frac{\alpha}{1-\alpha}\right]^{-\alpha} =$$

$$= \left(\frac{W_t}{A_t}\right)^{1-\alpha}\left(R_t^k\right)^{\alpha}\frac{1}{\alpha^{\alpha}\left(1-\alpha\right)^{1-\alpha}} \tag{B.9}$$

Next, I show that, given the duality property of the production function, the nominal marginal cost, MC_t, equals the nominal average variable cost, AVC_t. Specifically, substitute the relative demand for capital (B.8) into the aggregate production function and explicit it for labor $L_t(i)$, in order to obtain the following expression

$$L_t(i) = Y_t(i)A_t^{-(1-\alpha)}\left(\frac{W_t}{R_t^k}\frac{\alpha}{1-\alpha}\right)^{-\alpha} + A_t^{-(1-\alpha)}\left(\frac{W_t}{R_t^k}\frac{\alpha}{1-\alpha}\right)^{-\alpha}A_t\Upsilon_t^{\frac{\alpha}{1-\alpha}}F$$

Then, using the above expression together with (B.8) inside the total cost function $TC_t = W_tL_t(i) + R_t^kK_t(i)$ it is obtained

$$TC_t = \left(\frac{W_t}{A_t}\right)^{1-\alpha} \left(R_t^k\right)^{\alpha} \frac{1}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}} Y_t(i) + \left(\frac{W_t}{A_t}\right)^{1-\alpha} \left(R_t^k\right)^{\alpha} \frac{1}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}} A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}} F$$

where the first term on the right-hand side of the equation represents the total variable cost, which divided by $Y_t(i)$ gives the average variable cost clearly equal to the nominal marginal cost MC_t, given in equation (B.9).

Next, I derive the optimality condition of the fraction $1-\xi_p$ of firms that are allowed to choose their optimal price level in a given period. Note first that the remaining fraction ξ_p of firms that cannot optimally choose their price will reset it accordingly to the following indexation rule

$$P_t(i) = P_{t-1}(i)\pi_{t-1}^{\iota_p}\pi^{1-\iota_p}$$

where π is the level of inflation in steady state. Hence, inflation can be defined as follows

$$\pi_t = \frac{P_t(i)}{P_{t-1}(i)} = \pi_{t-1}^{\iota_p}\pi^{1-\iota_p}$$

Re-expressing the above equation for the infinite sum I obtain the total inflation generated from period t to period $t+s$, $\pi_{t,t+s}$

$$\pi_{t,t+s} = \frac{P_{t+s}(i)}{P_t(i)} = \prod_{j=0}^{s} \pi_{t+j-1}^{\iota_p}\pi^{1-\iota_p}$$

Hence, as far as the average variable cost equals the nominal marginal cost, I can express the present discounted value of future profits as follows

$$E_t \sum_{s=0}^{\infty} \xi_p^s \beta^s \frac{\Lambda_{t+s}}{\Lambda_t} \left[\tilde{P}_t(i) \left(\prod_{j=0}^{s} \pi_{t+j-1}^{\iota_p}\pi^{1-\iota_p} \right) Y_{t+s}(i) - W_{t+s}L_{t+s}(i) - R_{t+s}^k K_{t+s}(i) \right]$$

$$E_t \sum_{s=0}^{\infty} \xi_p^s \beta^s \frac{\Lambda_{t+s}}{\Lambda_t} \left[\tilde{P}_t(i) \left(\prod_{j=0}^{s} \pi_{t+j-1}^{\iota_p}\pi^{1-\iota_p} \right) - \frac{W_{t+s}L_{t+s}(i) - R_{t+s}^k K_{t+s}(i)}{Y_{t+s}(i)} \right] Y_{t+s}(i)$$

$$E_t \sum_{s=0}^{\infty} \xi_p^s \beta^s \frac{\Lambda_{t+s}}{\Lambda_t} \left[\tilde{P}_t(i) \left(\prod_{j=0}^{s} \pi_{t+j-1}^{\iota_p}\pi^{1-\iota_p} \right) - AVC_{t+s} \right] Y_{t+s}(i)$$

$$E_t \sum_{s=0}^{\infty} \xi_p^s \beta^s \frac{\Lambda_{t+s}}{\Lambda_t} \left[\tilde{P}_t(i) \left(\prod_{j=0}^{s} \pi_{t+j-1}^{\iota_p}\pi^{1-\iota_p} \right) - MC_{t+s} \right] Y_{t+s}(i)$$

The latter expression is, then, maximized with respect to the optimal price $\tilde{P}_t(i)$ subject to the optimal demand for intermediate good given in problem setting (P3). Namely, derive the FOC of the following Lagrangian equation, as follows

$$\mathcal{L} = E_t \sum_{s=0}^{\infty} \xi_p^s \beta^s \frac{\Lambda_{t+s}}{\Lambda_t} \left[\tilde{P}_t(i) \left(\prod_{j=0}^{s} \pi_{t+j-1}^{\iota_p} \pi^{1-\iota_p} \right) - MC_{t+s} \right] \left[\frac{\tilde{P}_t(i)}{P_{t+s}} \pi_{t,t+s} \right]^{-\frac{1+\lambda_{p,t+s}}{\lambda_{p,t+s}}} Y_{t+s} \tag{B.10}$$

$$\frac{\partial \mathcal{L}}{\partial \tilde{P}_t(i)} = E_t \sum_{s=0}^{\infty} \xi_p^s \beta^s \frac{\Lambda_{t+s}}{\Lambda_t} \left[-\frac{1}{\lambda_{p,t+s}} \tilde{P}_t(i)^{-\frac{1+\lambda_{p,t+s}}{\lambda_{p,t+s}}} \pi_{t,t+s} + \right.$$

$$\left. - MC_{t+s} \left(-\frac{1+\lambda_{p,t+s}}{\lambda_{p,t+s}} \right) \tilde{P}_t(i)^{-\frac{1+\lambda_{p,t+s}}{\lambda_{p,t+s}}-1} \right] \left(\frac{\pi_{t,t+s}}{P_{t+s}} \right)^{-\frac{1+\lambda_{p,t+s}}{\lambda_{p,t+s}}} Y_{t+s}$$

$$E_t \sum_{s=0}^{\infty} \xi_p^s \beta^s \Lambda_{t+s} \left[\tilde{P}_t(i) \pi_{t,t+s} - (1+\lambda_{p,t+s}) MC_{t+s} \right] \tilde{Y}_{t+s}(i) = 0 \tag{B.11}$$

Equation (B.11) describes the optimal price setting condition of the firms allowed to choose their price level. Recalling the optimal price function for the intermediate good i in (B.4) and the indexation rule common to the firms that are not optimizing for the price, the aggregate price index is defined as follows

$$P_t = \left[\xi_p \left(P_{t-1} \pi_{t-1}^{\iota_p} \pi^{1-\iota_p} \right)^{-\frac{1}{\lambda_{p,t}}} + (1-\xi_p) \tilde{P}_t^{-\frac{1}{\lambda_{p,t}}} \right]^{-\lambda_{p,t}} \tag{B.12}$$

Maintenance good sector

In order to calculate the optimality conditions in this sector I express the Lagrangian of the problem setting (P4), as follows

$$\mathcal{L} = E_t \sum_{s=0}^{\infty} \beta^s \Lambda_{t+s} \left\{ P_{t+s}^m d_{t+s} Y_{t+s}^m \left[1 - f\left(\frac{Y_{t+s}^m}{Y_{t+s-1}^m} \right) \right] - P_{t+s} Y_{t+s}^m \right\} \tag{B.13}$$

The FOC with respect to Y_t^m is computed as follows

$$\frac{\partial \mathcal{L}}{\partial Y_t^m} = -\Lambda_t P_t + \Lambda_t P_t^m d_t \left[1 - f\left(\frac{Y_t^m}{Y_{t-1}^m} \right) - \frac{Y_t^m}{Y_{t-1}^m} f'\left(\frac{Y_t^m}{Y_{t-1}^m} \right) \right] + \beta E_t \left\{ \Lambda_{t+1} d_{t+1} P_{t+1}^m \left(\frac{Y_{t+1}^m}{Y_t^m} \right)^2 f'\left(\frac{Y_{t+1}^m}{Y_t^m} \right) \right\}$$

$$\Lambda_t P_t = \Lambda_t P_t^m d_t \left[1 - f\left(\frac{Y_t^m}{Y_{t-1}^m} \right) - \frac{Y_t^m}{Y_{t-1}^m} f'\left(\frac{Y_t^m}{Y_{t-1}^m} \right) \right] + \beta E_t \left\{ \Lambda_{t+1} d_{t+1} P_{t+1}^m \left(\frac{Y_{t+1}^m}{Y_t^m} \right)^2 f'\left(\frac{Y_{t+1}^m}{Y_t^m} \right) \right\} \tag{B.14}$$

Equation (B.14) defines the optimal condition for maintenance goods supply.

Investment good sector

Firms producing investment goods are assumed to maximize with respect to the fraction of final good, Y_t^I, which is used for the production of investment goods. Their profit function, Π_t^I, is given by

$$\Pi_t^I = P_t^I \Upsilon_t Y_t^I - P_t Y_t^I$$

which is obtained simply substituting the technology for production of investment goods into the revenue function in problem setting (P5). Thus, the FOC is derived as follows

$$\frac{\partial \Pi_t^I}{\partial Y_t^I} = P_t^I \Upsilon_t - P_t$$

$$\frac{P_t^I}{P_t} = \Upsilon_t^{-1}$$

where the latter expression defines the relative price of investment as the inverse of the IST progress, Υ_t.

Capital good sector

Given the problem setting (P6) of the firms producing capital goods, I maximize the following expression of the discounted stream of profits with respect to I_t

$$\max_{I_t} \quad E_t \sum_{s=0}^{\infty} \beta^s \Lambda_{t+s} \left[P_{t+s}^k \mu_{t+s} \left[1 - S\left(\frac{I_{t+s}}{I_{t+s-1}} \right) \right] I_{t+s} - P_{t+s}^I I_{t+s} \right]$$

The following passages describe the analytical derivation of the FOC with respect to I_t

$$E_t \sum_{s=0}^{\infty} \beta^s \Lambda_{t+s} \left[P_{t+s}^k \mu_{t+s} - P_{t+s}^k \mu_{t+s} S\left(\frac{I_{t+s}}{I_{t+s-1}} \right) - P_{t+s}^k \mu_{t+s} \frac{I_{t+s}}{I_{t+s-1}} S'\left(\frac{I_{t+s}}{I_{t+s-1}} \right) - P_{t+s}^I \right] +$$
$$+ E_t \sum_{s=0}^{\infty} \beta^{s+1} \Lambda_{t+s+1} P_{t+s+1}^k \mu_{t+s+1} \left(\frac{I_{t+s+1}}{I_{t+s}} \right)^2 S'\left(\frac{I_{t+s+1}}{I_{t+s}} \right) = 0$$

$$\Lambda_t P_t^k \mu_t \left[1 - S\left(\frac{I_t}{I_{t-1}} \right) - \frac{I_t}{I_{t-1}} S'\left(\frac{I_t}{I_{t-1}} \right) \right] - \Lambda_t P_t^I + \beta E_t \left\{ \Lambda_{t+1} P_{t+1}^k \mu_{t+1} \left(\frac{I_{t+1}}{I_t} \right)^2 S'\left(\frac{I_{t+1}}{I_t} \right) \right\} = 0$$

$$P_t^I = P_t^k \mu_t \left[1 - S\left(\frac{I_t}{I_{t-1}} \right) - \frac{I_t}{I_{t-1}} S'\left(\frac{I_t}{I_{t-1}} \right) \right] + \beta E_t \left\{ \frac{\Lambda_{t+1}}{\Lambda_t} P_{t+1}^k \mu_{t+1} \left(\frac{I_{t+1}}{I_t} \right)^2 S'\left(\frac{I_{t+1}}{I_t} \right) \right\} \tag{B.15}$$

Equation (B.15) defines the optimal supply curve for capital goods.
When there are no adjustment costs of investment, i.e. $S = S' = 0$, equation (B.15) reduces to $q_t = \mu_t^{-1}$, that is, the relative price of capital with respect to investment, which defines the Tobin's q, is equal to the inverse of the shock to marginal efficiency of investment, μ_t.

The first order condition with respect to new capital, i_t^k, delivers an expression for the price of new capital as a ratio between the shadow value of installed new capital, Φ_t, and the households Lagrangian multiplier

$$P_t^k = \frac{\Phi_t}{\Lambda_t}$$

substituting the latter expression back into the equation (B.15) gives the following optimal condition for new investment

$$\Lambda_t P_t^I = \Phi_t \mu_t \left[1 - S\left(\frac{I_t}{I_{t-1}}\right) - \frac{I_t}{I_{t-1}} S'\left(\frac{I_t}{I_{t-1}}\right)\right] + \beta E_t \left\{ \Phi_{t+1}\mu_{t+1} \left(\frac{I_{t+1}}{I_t}\right)^2 S'\left(\frac{I_{t+1}}{I_t}\right)\right\} \tag{B.16}$$

Recalling that the Tobin's q is defined as the relative marginal value of installed capital with respect to investment, that is

$$q_t = \frac{\Phi_t}{P_t^I \Lambda_t}$$

where the denominator represents the replacement investment cost, the price for new capital becomes

$$P_t^k = q_t P_t \Upsilon_t^{-1}$$

According to the latter expression, the relative price of new capital is given by the Tobin's q times the inverse of investment specific technology progress. In the absence of investment adjustment costs, instead, the relative price of new capital with respect to consumption can be re-expressed as

$$\frac{P_t^k}{P_t} = \Upsilon_t^{-1}\mu_t^{-1}$$

Hence, a positive shock to new investment makes the relative price of new capital to decline. The same happens with respect to the marginal efficiency of investment shock, μ, when investment adjustment costs are excluded.

Employment agencies sector

Recalling the problem setting (P7) the Lagrangian is written as follows

$$\mathcal{L} = W_t L_t - \int_0^1 W_t(j) L_t(j) dj + \Lambda_t \left\{ \left[\int_0^1 L_t(j)^{\frac{1}{1+\lambda_{w,t}}} dj\right]^{1+\lambda_{w,t}} - L_t \right\} \tag{B.17}$$

The FOC with respect to homogeneous labor, L_t, gives $W_t = \Lambda_t$, while, the FOC with respect to $L_t(j)$ is calculated as follows

$$\frac{\partial \mathcal{L}}{\partial L_t(i)} = -W_t(j) + \Lambda_t \left[\int_0^1 L_t(j)^{\frac{1}{1+\lambda_{w,t}}} dj\right]^{1+\lambda_{w,t}-1} L_t(j)^{\frac{1}{1+\lambda_{w,t}}-1}$$

$$W_t(j) = W_t L_t(j)^{-\frac{\lambda_{w,t}}{1+\lambda_{w,t}}} \left\{ \left[\int_0^1 L_t(j)^{\frac{1}{1+\lambda_{w,t}}} dj\right]^{1+\lambda_{w,t}} \right\}^{\frac{\lambda_{w,t}}{1+\lambda_{w,t}}}$$

$$L_t(j) = \left[\frac{W_t(j)}{W_t}\right]^{-\frac{1+\lambda_{w,t}}{\lambda_{w,t}}} L_t \tag{B.18}$$

Expression (B.18) defines the optimal demand function for heterogeneous labor, which is supplied by the households, and is transformed by the employment agencies into homogeneous labor purchased, thereafter, by the intermediate goods producers at the cost W_t.
Starting with the zero profit condition, which implies that $W_t(j)L_t(j) = W_tL_t$, and substituting for equation (B.18), the following passages derive the optimal aggregate wage function for homogeneous labor, W_t,

$$W_tL_t - \int_0^1 W_t(j)\left[\frac{W_t(j)}{W_t}\right]^{-\frac{1+\lambda_{w,t}}{\lambda_{w,t}}} L_t dj = 0$$

$$W_t - W_t^{\frac{1+\lambda_{w,t}}{\lambda_{w,t}}} \int_0^1 W_t(j)W_t(j)^{-\frac{1+\lambda_{w,t}}{\lambda_{w,t}}} dj = 0$$

$$W_t = \left[\int_0^1 W_t(j)^{-\frac{1}{\lambda_{w,t}}} dj\right]^{-\lambda_{w,t}} \tag{B.19}$$

Households
Given the problem setting (P8), I define the Lagrangian as follows

$$\begin{aligned}
\mathcal{L} = E_t \sum_{s=0}^{\infty} \beta^s \Bigg\{ & b_{t+s}\left[\log\left(C_{t+s} - hC_{t+s-1}\right) - \varphi \frac{L_{t+s}(j)^{1+\nu}}{1+\nu}\right] + \\
& + \Lambda_{t+s}\Bigg[R_{t+s-1}B_{t+s-1} + Q_{t+s}(j) + \Pi_{t+s} + W_{t+s}(j)L_{t+s}(j) + R^k_{t+s}u_{t+s}\bar{K}_{t+s-1} + \\
& - \frac{P_{t+s}}{\Upsilon_{t+s}}a(u_{t+s})\bar{K}_{t+s-1} - P_{t+s}C_{t+s} - P^k_{t+s}i^k_{t+s} - P^m_{t+s}M_{t+s} - T_{t+s} - B_{t+s}\Bigg] + \\
& + \Phi_{t+s}\left[\left[1 - \Upsilon^{-\sigma}_{t+s-1}\left(\zeta u^{\eta}_{t+s}\left(\frac{M_{t+s}}{\bar{K}_{t+s-1}}\right)^{-\sigma} + \Upsilon^{\sigma}_{t+s}\bar{\delta}\right)\right]\bar{K}_{t+s-1} + i^k_{t+s} - \bar{K}_{t+s}\right] + \\
& + \Gamma_{t+s}\left[\tau\Upsilon^{-1}_{t+s}u_{t+s}\bar{K}_{t+s-1} + A_{t+s}\Upsilon^{\frac{\alpha}{1-\alpha}}_{t+s}\bar{M} - M_{t+s}\right]\Bigg\}
\end{aligned} \tag{B.20}$$

Next, I calculate the FOC with respect to current consumption, C_t, as follows

$$\frac{\partial \mathcal{L}}{\partial C_t} = \frac{b_t}{C_t - hC_{t-1}} - \Lambda_tP_t - \beta hE_t\left\{\frac{b_{t+1}}{C_{t+1} - hC_t}\right\}$$

$$\Lambda_tP_t = \frac{b_t}{C_t - hC_{t-1}} - \beta hE_t\left\{\frac{b_{t+1}}{C_{t+1} - hC_t}\right\} \tag{B.21}$$

which yields the expression for the marginal utility of nominal income.

The FOC with respect to current bonds holdings, B_t, defines the consumption Euler equation, that is

$$\frac{\partial \mathcal{L}}{\partial B_t} = -\Lambda_t + \beta R_t E_t \Big\{ \Lambda_{t+1} \Big\}$$

$$\Lambda_t = \beta R_t E_t \Big\{ \Lambda_{t+1} \Big\} \tag{B.22}$$

or, similarly

$$1 = E_t \left[\beta \frac{\Lambda_{t+1} P_{t+1}}{\Lambda_t P_t} R_t \frac{P_t}{P_{t+1}} \right]$$

where, the expression $\beta \frac{\Lambda_{t+1} P_{t+1}}{\Lambda_t P_t}$ defines the real stochastic discount factor.

The FOC with respect to current labor, $L_t(j)$, is

$$\frac{\partial \mathcal{L}}{\partial L_t} = -b_t \varphi L_t(j)^{\nu} + \Lambda_t W_t(j)$$

$$\Lambda_t W_t(j) = b_t \varphi L_t(j)^{\nu} \tag{B.23}$$

where the latter expression equates wages in terms of the shadow price for consumption to the marginal utility of labor.

The FOC with respect to new capital, i_t^k, is

$$\frac{\partial \mathcal{L}}{\partial i_t^k} = -\Lambda_t P_t^k + \Phi_t$$

$$P_t^k = \Phi_t / \Lambda_t \tag{B.24}$$

which says that, in equilibrium, the price of capital equals the relative shadow value of capital with respect to the shadow value of consumption. Substitute equation (B.24) into the optimal condition for capital supply (B.15) in order to obtain the following expression

$$\Lambda_t P_t^I = \Phi_t \mu_t \left[1 - S\left(\frac{I_t}{I_{t-1}} \right) - \frac{I_t}{I_{t-1}} S'\left(\frac{I_t}{I_{t-1}} \right) \right] + \beta E_t \Big\{ \Phi_{t+1} \mu_{t+1} \left(\frac{I_{t+1}}{I_t} \right)^2 S'\left(\frac{I_{t+1}}{I_t} \right) \Big\} \tag{B.25}$$

The FOC with respect to maintenance, M_t, is derived as follows

$$\frac{\partial \mathcal{L}}{\partial M_t} = -\Lambda_t P_t^m - \Phi_t \Upsilon_{t-1}^{-\sigma} \bar{K}_{t-1} \zeta u_t^{\eta} (-\sigma) \left(\frac{M_t}{\bar{K}_{t-1}} \right)^{-\sigma-1} \frac{1}{\bar{K}_{t-1}} - \Gamma_t$$

$$\Lambda_t P_t^m = \sigma \zeta \Phi_t \Upsilon_{t-1}^{-\sigma} u_t^{\eta} \left(\frac{M_t}{\bar{K}_{t-1}} \right)^{-\sigma-1} - \Gamma_t$$

which, in terms of gross depreciation rate, is equivalent to

$$\Lambda_t P_t^m = \sigma \Phi_t \Upsilon_{t-1}^{-\sigma} \left(\frac{M_t}{\bar{K}_{t-1}}\right)^{-1} \left(D_t - \Upsilon_t^{\sigma} \bar{\delta}\right) - \Gamma_t \tag{B.26}$$

In equilibrium, demand for maintenance goods is negatively related to its price, P_t^m, and positively related to the gross depreciation rate evaluated at the shadow value of capital. Moreover, as far as I have imposed the following conditions on parameters $\sigma > 0$, $0 < \zeta < 1$, and $\eta > 1$, the former equation also suggests that maintenance demand is positively related to capital utilization rate, u_t. So, at optimum, the price for maintenance must be equal to the marginal benefit accruing from the capital depreciation rate given an increase of one unit of maintenance evaluated at the relative shadow value of new capital minus the relative shadow value of maintenance.
Substituting equation (B.26) into (B.14) I define the following optimal condition for maintenance goods

$$\begin{aligned} \Lambda_t P_t = & \left[\sigma \Phi_t \Upsilon_{t-1}^{-\sigma} \left(\frac{M_t}{\bar{K}_{t-1}}\right)^{-1} \left(D_t - \Upsilon_t^{\sigma} \bar{\delta}\right) - \Gamma_t\right] d_t \left[1 - f\left(\frac{Y_t^m}{Y_{t-1}^m}\right) - \frac{Y_t^m}{Y_{t-1}^m} f'\left(\frac{Y_t^m}{Y_{t-1}^m}\right)\right] + \\ & + \beta E_t \Bigg\{ \left[\sigma \Phi_{t+1} \Upsilon_t^{-\sigma} \left(\frac{M_{t+1}}{\bar{K}_t}\right)^{-1} \left(D_{t+1} - \Upsilon_{t+1}^{\sigma} \bar{\delta}\right) - \Gamma_{t+1}\right] d_{t+1} \left(\frac{Y_{t+1}^m}{Y_t^m}\right)^2 f'\left(\frac{Y_{t+1}^m}{Y_t^m}\right) \Bigg\} \end{aligned} \tag{B.27}$$

A representative household, further more, chooses the level of capital utilization rate, u_t. The optimal condition for utilization is derived as follows

$$\frac{\partial \mathcal{L}}{\partial u_t} = \Lambda_t R_t^k \bar{K}_{t-1} - \Lambda_t P_t \Upsilon_t^{-1} a'(u_t) \bar{K}_{t-1} - \zeta \eta \Phi_t \Upsilon_{t-1}^{-\sigma} \bar{K}_{t-1} \left(\frac{M_t}{\bar{K}_{t-1}}\right)^{-\sigma} u_t^{\eta-1} + \tau \Gamma_t \Upsilon_t^{-1}$$

$$\Lambda_t R_t^k = \Lambda_t P_t \Upsilon_t^{-1} a'(u_t) + \zeta \eta \Phi_t \Upsilon_{t-1}^{-\sigma} \left(\frac{M_t}{\bar{K}_{t-1}}\right)^{-\sigma} u_t^{\eta-1} + \tau \Gamma_t \Upsilon_t^{-1}$$

Substituting back the assumed depreciation rate function, the latter expression becomes

$$\Lambda_t R_t^k = \Lambda_t P_t \Upsilon_t^{-1} a'(u_t) + \eta \Phi_t \Upsilon_{t-1}^{-\sigma} u_t^{-1} \left(D_t - \Upsilon_t^{\sigma} \bar{\delta}\right) + \tau \Gamma_t \Upsilon_t^{-1} \tag{B.28}$$

According to the above expression, the rental price of capital, at optimum, must equate the marginal value of capital utilization costs evaluated at the current shadow value of consumption plus the cost deriving from a higher capital depreciation rate due to a marginal increase in the utilization rate evaluated at the relative shadow value of new capital net of the marginal benefits accruing from a higher maintenance activity evaluated at the relative shadow value of maintenance.

Next, I derive the FOC with respect to capital stock at time t, $\bar{K}_t$, as follows

$$\frac{\partial \mathcal{L}}{\partial \bar{K}_t} = \beta\Lambda_{t+1}R^k_{t+1}u_{t+1} - \beta\Lambda_{t+1}P_{t+1}\Upsilon^{-1}_{t+1}a\left(u_{t+1}\right) - \Phi_t + \beta\Phi_{t+1}\left(1 - \Upsilon^{-\sigma}_t D_{t+1}\right) +$$

$$+ \beta\tau\Gamma_{t+1}\Upsilon^{-1}_{t+1}u_{t+1} - \beta\Phi_{t+1}\Upsilon^{-\sigma}_t \bar{K}_t\zeta\left(-\sigma\right)u^{\eta}_{t+1}\left(\frac{M_{t+1}}{\bar{K}_t}\right)^{-\sigma-1}\left(-\frac{M_{t+1}}{\bar{K}^2_t}\right)$$

Rearranging and substituting for the equation of depreciation in the latter term on the right-hand side of the above equation, I obtain the following optimal expression for the demand of capital stock

$$\Phi_t = \beta E_t\Bigg\{\Lambda_{t+1}\left[R^k_{t+1}u_{t+1} - P_{t+1}\Upsilon^{-1}_{t+1}a\left(u_{t+1}\right)\right] + \Phi_{t+1}\left(1 - \Upsilon^{-\sigma}_t D_{t+1}\right) +$$

$$+ \tau\Gamma_{t+1}\Upsilon^{-1}_{t+1}u_{t+1} - \sigma\Phi_{t+1}\Upsilon^{-\sigma}_t\left(D_{t+1} - \Upsilon^{\sigma}_{t+1}\bar{\delta}\right)\Bigg\} \tag{B.29}$$

where Φ_t is the shadow value of the new installed capital. The last two terms on the right-hand side of the above equation capture the effects of the maintenance activity and of the variable depreciation rate, respectively. The higher the expected maintenance the lower the expected depreciation rate and the higher the current shadow price of capital. On the contrary, the higher the expected depreciation rate the lower the current shadow price. The effect of the expected capital utilization rate, instead, is ambiguous. In fact, it will tend to increase the current value of the capital shadow price through higher maintenance activities but, at the same time, it will decrease the shadow price because of a higher expected depreciation.

Finally, the fraction $1 - \xi_w$ of representative households optimally set the level of their wages by optimizing with respect to $\tilde{W}_t(j)$ the following expression of the discounted stream of future earnings, given in the problem setting (P9)

$$\max_{\tilde{W}_t(j)} \quad E_t\sum_{s=0}^{\infty}\beta^s\xi^s_w\Bigg\{\Lambda_{t+s}\tilde{W}_t(j)^{1-\frac{1+\lambda_{w,t}}{\lambda_{w,t}}}\left(\frac{\pi^w_{t,t+s}}{W_{t+s}}\right)^{-\frac{1+\lambda_{w,t}}{\lambda_{w,t}}}L_{t+s}\pi^w_{t,t+s} +$$

$$- b_{t+s}\frac{\varphi}{1+\nu}\tilde{W}_t(j)^{-\frac{1+\lambda_{w,t}}{\lambda_{w,t}}(1+\nu)}\left[\left(\frac{\pi^w_{t,t+s}}{W_{t+s}}\right)^{-\frac{1+\lambda_{w,t}}{\lambda_{w,t}}}L_{t+s}\right]^{1+\nu}\Bigg\}$$

The FOC is, therefore

$$E_t\sum_{s=0}^{\infty}\beta^s\xi^s_w\Bigg\{-\frac{1}{\lambda_{w,t}}\Lambda_{t+s}\tilde{W}_t(j)^{-\frac{1+\lambda_{w,t}}{\lambda_{w,t}}}\left(\frac{\pi^w_{t,t+s}}{W_{t+s}}\right)^{-\frac{1+\lambda_{w,t}}{\lambda_{w,t}}}L_{t+s}\pi^w_{t,t+s} +$$

$$- b_{t+s}\varphi\left(-\frac{1+\lambda_{w,t}}{\lambda_{w,t}}\right)\tilde{W}_t(j)^{-\frac{1+\lambda_{w,t}}{\lambda_{w,t}}(1+\nu)-1}\left[\left(\frac{\pi^w_{t,t+s}}{W_{t+s}}\right)^{-\frac{1+\lambda_{w,t}}{\lambda_{w,t}}}L_{t+s}\right]^{1+\nu}\Bigg\} = 0$$

Which, rearranging, gives the following optimal wage setting condition

$$E_t \sum_{s=0}^{\infty} \xi_w^s \beta^s \Lambda_{t+s} \tilde{L}_{t+s}(j) \left[\pi_{t,t+s}^w \tilde{W}_t(j) - b_{t+s}\varphi \left(1 + \lambda_{w,t+s}\right) \frac{\tilde{L}_{t+s}(j)^\nu}{\Lambda_{t+s}} \right] = 0 \tag{B.30}$$

Given the indexation rule followed by the fraction ξ_w of households, who does not optimize for wages, as

$$W_t(j) = W_{t-1}(j) \left(\pi_{t-1} e^{z_{t-1} + \frac{\alpha}{1-\alpha} v_{t-1}}\right)^{\iota_w} \left(\pi e^{\gamma_z + \frac{\alpha}{1-\alpha}\gamma_v}\right)^{1-\iota_w}$$

The aggregate wage index is straightforward

$$W_t = \left(\xi_w \left[W_{t-1} \left(\pi_{t-1} e^{z_{t-1} + \frac{\alpha}{1-\alpha} v_{t-1}}\right)^{\iota_w} \left(\pi e^{\gamma_z + \frac{\alpha}{1-\alpha}\gamma_v}\right)^{1-\iota_w} \right]^{-\frac{1}{\lambda_{w,t}}} + (1-\xi_w) \tilde{W}_t(j)^{-\frac{1}{\lambda_{w,t}}} \right)^{-\lambda_{w,t}} \tag{B.31}$$

The model is closed by the expressions for the aggregate resource constraint and for the actual GDP, X_t, which are given by, respectively

$$C_t + \Upsilon_t^{-1} I_t + \Upsilon_t^{-1} a(u_t) \bar{K}_{t-1} + \tilde{M}_t = \frac{Y_t}{g_t} \tag{B.32}$$

$$X_t = \left(1 - \frac{1}{g_t}\right) Y_t + C_t + \Upsilon_t^{-1} I_t + \tilde{M}_t \tag{B.33}$$

Following Justiniano et al. (2011), the market clearing conditions are obtained from the aggregation of the households' and Government budget constraints combined with the zero profit conditions in the production sectors of final good, capital-good, investment-good, maintenance services and in the employment agencies sector. The functional form of the actual GDP, instead, is obtained by substitution of the definition for public spendings into the aggregate resource constraint, given that the actual GDP is defined as $X_t = G_t + C_t + \Upsilon_t^{-1} I_t + \tilde{M}_t$.

One-sector environment

Recalling the definition of real maintenance and real investment in terms of consumption as $\tilde{M}_t = (P_t^m / P_t) M_t$ and $\tilde{I}_t = (P_t^I / P_t) I_t$, respectively, the decentralized model can be reduced to a one sector model, as follows

$$P_t C_t + P_t \tilde{I}_t + P_t \tilde{M}_t + T_t + B_t = R_{t-1} B_{t-1} + Q_t(j) + \Pi_t + W_t(j) L_t(j) + R_t^k u_t \bar{K}_{t-1} - \frac{P_t}{\Upsilon_t} a(u_t) \bar{K}_{t-1}$$

$$\bar{K}_t = (1 - \Upsilon_{t-1}^{-\sigma} D_t) \bar{K}_{t-1} + \mu_t \Upsilon_t \left(1 - S_t\right) \tilde{I}_t$$

$$D_t = \zeta u_t^\eta \left(\frac{d_t \tilde{M}_t \left[1 - f\left(\tilde{M}_t / \tilde{M}_{t-1}\right)\right]}{K_{t-1}} \right)^{-\sigma} + \Upsilon_t^\sigma \bar{\delta}$$

$$d_t \tilde{M}_t \left[1 - f\left(\tilde{M}_t / \tilde{M}_{t-1}\right)\right] = \tau \Upsilon_t^{-1} u_t \bar{K}_{t-1} + A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}} \bar{M}$$

In this case the law of motion of capital is subject to the composite shock, $\mu_t \Upsilon_t$, which is used to be considered in the related literature as the investment-specific technology shock[27]. In my model, therefore, according to Justiniano et al. (2011), it is decomposed in two effects: the first one affects the efficiency of new investment (μ), and the latter one affects the production of new investment (Υ). Given the roles these shocks play in my decentralized economy, I interpret the IST shock, Υ, as the disembodied investment-specific technology progress and identify it with the inverse of the relative price of investment. For the estimation purposes the relative price of investment is treated as observable which, thus, pins down the evolution of IST. The MEI shock, μ, is interpreted as the embodied investment-specific technology progress, which explains the quality improvement of investment and therefore determines the rate of obsolescence of capital stock. Both of these concepts are strictly related with the economic depreciation rate, which accelerates when a higher quality capital is available on the market (obsolescence). On the contrary, it is shown by the present model that in response to a positive IST shock depreciation decreases as a consequence of other stronger indirect effects.

[27] See, for example, Smets and Wouters (2007)

C APPENDIX: Trends

In this section of the appendix I analytically derive the expressions of the trends of the main endogenous variables. I will denote by g the growth rates of the respective endogenous variables of the model.

Intermediate good production function

Recall the aggregate production technology of the intermediate good, that is

$$Y_t(i) = max\Big\{A_t^{1-\alpha} K_t^{\alpha} L_t^{1-\alpha} - A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}} F; 0\Big\}$$

Assume a representative firm decides not to produce then, the aggregate production technology becomes

$$A_t^{1-\alpha} K_t^{\alpha} L_t^{1-\alpha} - A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}} F = 0$$

Differentiating the latter expression with respect to time and assuming that hours worked, L_t, exhibit no growth over time, I obtain

$$(1-\alpha) A_t^{1-\alpha-1} K_t^{\alpha} L_t^{1-\alpha} \frac{\partial A_t}{\partial t} + \alpha A_t^{1-\alpha} K_t^{\alpha-1} L_t^{1-\alpha} \frac{\partial K_t}{\partial t} - \Upsilon_t^{\frac{\alpha}{1-\alpha}} F \frac{\partial A_t}{\partial t} - \frac{\alpha}{1-\alpha} A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}-1} F \frac{\partial \Upsilon_t}{\partial t} = 0$$

$$(1-\alpha) A_t^{1-\alpha} K_t^{\alpha} L_t^{1-\alpha} g_A + \alpha A_t^{1-\alpha} K_t^{\alpha} L_t^{1-\alpha} g_K - A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}} F g_A - \frac{\alpha}{1-\alpha} A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}} F g_\Upsilon = 0$$

$$A_t^{1-\alpha} K_t^{\alpha} L_t^{1-\alpha} \left[(1-\alpha) g_A + \alpha g_K\right] - A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}} F \left[g_A + \frac{\alpha}{1-\alpha} g_\Upsilon\right] = 0$$

$$(1-\alpha) g_A + \alpha g_K = g_A + \frac{\alpha}{1-\alpha} g_\Upsilon$$

$$g_K = g_A + \left(\frac{\alpha}{1-\alpha} + 1\right) g_\Upsilon \tag{C.1}$$

Thus, the growth rate of effective capital, g_K, is given by a linear combination of the growth rates of neutral technology progress, g_A, and the investment- specific technology progress, g_Υ, which, in steady state, are given by γ_z and γ_v, respectively.

On the contrary, when $Y_t(i) > 0$, differentiating with respect to time I obtain

$$\frac{\partial Y(i)_t}{\partial t} = (1-\alpha) A_t^{1-\alpha-1} K_t^{\alpha} L_t^{1-\alpha} \frac{\partial A_t}{\partial t} + \alpha A_t^{1-\alpha} K_t^{\alpha-1} L_t^{1-\alpha} \frac{\partial K_t}{\partial t} - \Upsilon_t^{\frac{\alpha}{1-\alpha}} F \frac{\partial A_t}{\partial t} - \frac{\alpha}{1-\alpha} A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}-1} F \frac{\partial \Upsilon_t}{\partial t}$$

$$Y_t(i) g_Y = A_t^{1-\alpha} K_t^{\alpha} L_t^{1-\alpha} \left[(1-\alpha) g_A + \alpha g_K\right] - A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}} F \left[g_A + \frac{\alpha}{1-\alpha} g_\Upsilon\right]$$

Using (C.1) in the above expression, it follows that

$$Y_t(i) g_Y = A_t^{1-\alpha} K_t^{\alpha} L_t^{1-\alpha} \left[g_A + \frac{\alpha}{1-\alpha} g_\Upsilon\right] - A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}} F \left[g_A + \frac{\alpha}{1-\alpha} g_\Upsilon\right]$$

$$g_Y = g_A + \frac{\alpha}{1-\alpha} g_\Upsilon \tag{C.2}$$

The growth rate of investment in efficiency units, I_t, can be found by exploiting the respective production technology

$$I_t = \Upsilon_t Y_t^I$$

$$\frac{\partial I_t}{\partial t} = Y_t^I \frac{\partial \Upsilon_t}{\partial t} + \Upsilon_t \frac{\partial Y_t^I}{\partial t}$$

$$I_t g_I = \Upsilon_t Y_t^I g_\Upsilon + \Upsilon_t Y_t^I g_Y$$

$$g_I = g_\Upsilon + g_Y$$

$$g_I = g_K = g_A + \left(\frac{\alpha}{1-\alpha} + 1\right) g_\Upsilon \tag{C.3}$$

Given the definition of capital utilization rate, and assuming that capital utilization rate, u_t, does not grow over time, it follows that

$$K_t = u_t \bar{K}_{t-1}$$

$$\frac{\partial K_t}{\partial t} = u_t \frac{\partial \bar{K}_{t-1}}{\partial t}$$

$$K_t g_K = u_t \bar{K}_{t-1} g_{\bar{K}}$$

$$g_K = g_{\bar{K}} = g_A + \left(\frac{\alpha}{1-\alpha} + 1\right) g_\Upsilon \tag{C.4}$$

Following the assumptions with respect to maintenance cost function I find the growth rate of maintenance as follows

$$M_t = \tau \Upsilon_t^{-1} u_t \bar{K}_{t-1} + A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}} \bar{M}$$

$$\frac{\partial M_t}{\partial t} = -\tau \Upsilon_t^{-2} u_t \bar{K}_{t-1} \frac{\partial \Upsilon_t}{\partial t} + \tau \Upsilon_t^{-1} u_t \frac{\partial \bar{K}_{t-1}}{\partial t} + \Upsilon_t^{\frac{\alpha}{1-\alpha}} \bar{M} \frac{\partial A_t}{\partial t} + \frac{\alpha}{1-\alpha} A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}-1} \bar{M} \frac{\partial \Upsilon_t}{\partial t}$$

$$M_t g_M = -\tau \Upsilon_t^{-1} u_t \bar{K}_{t-1} g_\Upsilon + \tau \Upsilon_t^{-1} u_t \bar{K}_{t-1} g_{\bar{K}} + A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}} \bar{M} g_A + \frac{\alpha}{1-\alpha} A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}} \bar{M} g_\Upsilon$$

$$g_M = g_Y = g_A + \frac{\alpha}{1-\alpha} g_\Upsilon \tag{C.5}$$

Recall the following functional form for the gross rate of depreciation, D_t,

$$D_t = \zeta u_t^\eta \left(\frac{M_t}{\bar{K}_{t-1}}\right)^{-\sigma} + \Upsilon_t^\sigma \bar{\delta}$$

By the total differentiation of the above expression I obtain

$$\frac{\partial D_t}{\partial t} = -\sigma\zeta u_t^{\eta}\left(\frac{M_t}{\bar{K}_{t-1}}\right)^{-\sigma}\left(\frac{1}{M_t}\frac{\partial M_t}{\partial t} - \frac{1}{\bar{K}_{t-1}}\frac{\partial \bar{K}_{t-1}}{\partial t}\right) + \sigma\bar{\delta}\Upsilon_t^{\sigma-1}\frac{\partial \Upsilon_t}{\partial t}$$

$$D_t g_\delta = \left[\zeta u_t^{\eta}\left(\frac{M_t}{\bar{K}_{t-1}}\right)^{-\sigma} + \Upsilon_t^{\sigma}\bar{\delta}\right]\sigma g_\gamma$$

$$g_\delta = \sigma g_\Upsilon \tag{C.6}$$

Therefore, equation (C.6) shows that the gross capital depreciation rate exhibits a trend, which follows the IST shock, Υ_t and it moreover depends on the sensibility of the depreciation function with respect to maintenance to capital ratio, σ.

I next find the growth rate of new investment goods, i_t^k, using the expression for capital accumulation

$$\bar{K}_t = \left(1 - \Upsilon_{t-1}^{-\sigma} D_t\right)\bar{K}_{t-1} + i_t^K$$

$$\frac{\partial \bar{K}_t}{\partial t} = \left(1 - \Upsilon_{t-1}^{-\sigma} D_t\right)\frac{\bar{K}_{t-1}}{\partial t} - \sigma\Upsilon_{t-1}^{-\sigma-1} D_t \bar{K}_{t-1}\frac{\partial \Upsilon_{t-1}}{\partial t} - \Upsilon_{t-1}^{-\sigma}\bar{K}_{t-1}\frac{\partial D_t}{\partial t} + \frac{\partial i_t^K}{\partial t}$$

$$\bar{K}_t g_{\bar{K}} = \left(1 - \Upsilon_{t-1}^{-\sigma} D_t\right)\bar{K}_{t-1} g_{\bar{K}} + \sigma\Upsilon_{t-1}^{-\sigma} D_t \bar{K}_{t-1} g_\Upsilon - \Upsilon_{t-1}^{-\sigma} D_t \bar{K}_{t-1} g_\delta + i_t^K g_{i^K}$$

$$\bar{K}_t g_{\bar{K}} = \left(1 - \Upsilon_{t-1}^{-\sigma} D_t\right)\bar{K}_{t-1} g_{\bar{K}} + i_t^K g_{i^K}$$

$$g_{i^K} = g_K = g_{\bar{K}} = g_I = g_A + \left(\frac{\alpha}{1-\alpha} + 1\right) g_\Upsilon \tag{C.7}$$

Recalling the aggregate resource constraint, I derive the growth rate of consumption as follows

$$C_t + \Upsilon_t^{-1} I_t + \Upsilon_t^{-1} a(u_t)\bar{K}_{t-1} = \frac{Y_t}{g_t}$$

where g_t is a stationary government spending shock.

$$\frac{\partial C_t}{\partial t} - \Upsilon_t^{-2} I_t \frac{\partial \Upsilon_t}{\partial t} + \Upsilon_t^{-1}\frac{\partial I_t}{\partial t} - \Upsilon_t^{-2} a(u_t)\bar{K}_{t-1}\frac{\partial \Upsilon_t}{\partial t} + \Upsilon_t^{-1} a(u_t)\frac{\partial \bar{K}_{t-1}}{\partial t} = \frac{1}{g_t}\frac{\partial Y_t}{\partial t}$$

$$C_t g_C - \Upsilon_t^{-1} I_t g_\Upsilon + \Upsilon_t^{-1} I_t g_I - \Upsilon_t^{-1} a(u_t)\bar{K}_{t-1} g_\Upsilon + \Upsilon_t^{-1} a(u_t)\bar{K}_{t-1} g_{\bar{K}} = \frac{1}{g_t} Y_t g_Y$$

$$C_t g_C - \Upsilon_t^{-1} I_t \left[g_\Upsilon - g_I\right] - \Upsilon_t^{-1} a(u_t)\bar{K}_{t-1}\left[g_\Upsilon - g_{\bar{K}}\right] = \frac{1}{g_t} Y_t g_Y$$

$$C_t g_C = \left[\frac{1}{g_t} Y_t - \Upsilon_t^{-1} I_t - \Upsilon_t^{-1} a(u_t)\bar{K}_{t-1}\right]\left(g_A + \frac{\alpha}{1-\alpha} g_\Upsilon\right)$$

$$g_C = g_M = g_Y = g_A + \frac{\alpha}{1-\alpha} g_\Upsilon \tag{C.8}$$

which implies that actual GDP, X_t, grows at the same rate

$$g_X = g_C = g_M = g_Y = g_A + \frac{\alpha}{1-\alpha} g_\Upsilon \tag{C.9}$$

D APPENDIX: Stationary equilibria and steady states

Given the normalization conditions obtained in the previous section of the appendix, I derive here the model equilibrium conditions in terms of stationary variables. The lower case variables represent the normalized stationary variables and those with no timing subsciption are the respective steady state values. The main model variables have been de-trended as follows

- $y_t = \frac{Y_t}{A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}}}$
- $x_t = \frac{X_t}{A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}}}$
- $c_t = \frac{C_t}{A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}}}$
- $\bar{k}_t = \frac{\bar{K}_t}{A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}+1}}$
- $k_t = \frac{K_t}{A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}+1}}$
- $i_t = \frac{I_t}{A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}+1}}$
- $m_t = \frac{M_t}{A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}}}$
- $w_t = \frac{W_t}{P_t A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}}}$
- $\rho_t = \frac{R_t^k}{P_t}$
- $s_t = \frac{MC_t}{P_t}$
- $\tilde{p}_t = \frac{\tilde{P}_t}{P_t}$
- $\pi_t = \frac{P_t}{P_{t-1}}$
- $\lambda_t = \Lambda_t P_t A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}}$
- $\phi_t = \Phi_t A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}+1}$

where s_t is the real marginal cost and ρ_t is the real return on capital. Moreover, the rate of capital depreciation is de-trended by the investment specific technology shock, that is $\delta_t = D_t/\Upsilon_t^{\sigma}$.

Intermediate good sector

Recalling the production function of the intermediate goods producers

$$Y_t(i) = A_t^{1-\alpha} K_t(i)^{\alpha} L_t(i)^{1-\alpha} - A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}} F$$

I obtain its stationary expression as follows

$$Y_t(i) \frac{A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}}}{A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}}} = A_t^{1-\alpha} \left[K_t(i) \frac{A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}+1}}{A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}+1}} \right]^{\alpha} L_t(i)^{1-\alpha} - A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}} F$$

$$y_t(i) A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}} = A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}} k_t(i)^{\alpha} L_t(i)^{1-\alpha} - A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}} F$$

$$y_t(i) = k_t(i)^{\alpha} L_t(i)^{1-\alpha} - F \tag{D.1}$$

The steady state relation of equation (D.1) is

$$y = k^{\alpha} L^{1-\alpha} - F$$

The zero profit condition implies

$$y - \rho k - wL = k^{\alpha} L^{1-\alpha} - F - \rho k - wL = 0$$

$$\left(\frac{k}{L}\right)^{\alpha} \frac{L}{L} - \frac{F}{L} - \rho \frac{k}{L} - w = 0$$

$$\left(\frac{k}{L}\right)^{\alpha} - \frac{F}{L} = \frac{y}{L} \tag{D.2}$$

The detrended optimal relative demand for capital given in (B.8) is obtained as follows

$$\frac{K_t}{L_t} \frac{A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}+1}}{A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}+1}} = \frac{\alpha}{1-\alpha} \frac{W_t}{R_t^k} \frac{P_t A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}}}{P_t A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}}} \frac{P_t \Upsilon_t^{-1}}{P_t \Upsilon_t^{-1}}$$

$$\frac{k_t}{L_t} A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}+1} = \frac{\alpha}{1-\alpha} \frac{w_t}{\rho_t} \frac{P_t A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}}}{P_t \Upsilon_t^{-1}}$$

$$\frac{k_t}{L_t} = \frac{\alpha}{1-\alpha} \frac{w_t}{\rho_t} \tag{D.3}$$

The latter expression, in steady state, becomes

$$\frac{k}{L} = \frac{\alpha}{1-\alpha} \frac{w}{\rho} \tag{D.4}$$

Given the expression for the nominal marginal cost (B.9), stationary real marginal cost is found as follows

$$MC_t \frac{P_t}{P_t} = \left(\frac{W_t}{A_t} \frac{P_t A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}}}{P_t A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}}}\right)^{1-\alpha} \left(R_t^k \frac{P_t \Upsilon_t^{-1}}{P_t \Upsilon_t^{-1}}\right)^{\alpha} \frac{1}{\alpha^{\alpha} (1-\alpha)^{1-\alpha}}$$

$$s_t P_t = \frac{1}{\alpha^{\alpha} (1-\alpha)^{1-\alpha}} w_t^{1-\alpha} \rho_t^{\alpha} P_t^{1-\alpha} \Upsilon_t^{\alpha} P_t^{\alpha} \Upsilon_t^{-\alpha}$$

$$s_t = \frac{1}{\alpha^{\alpha} (1-\alpha)^{1-\alpha}} w_t^{1-\alpha} \rho_t^{\alpha} \tag{D.5}$$

The steady state expression of the detrended real marginal cost is as follows

$$s = \frac{1}{\alpha^{\alpha} (1-\alpha)^{1-\alpha}} w^{1-\alpha} \rho^{\alpha} \tag{D.6}$$

The stationary equilibrium of the optimal price setting condition given in (B.11) is derived as follows

$$E_t \sum_{s=0}^{\infty} \xi_p^s \beta^s \Lambda_{t+s} \quad \frac{P_{t+s} A_{t+s} \Upsilon_{t+s}^{\frac{\alpha}{1-\alpha}}}{P_{t+s} A_{t+s} \Upsilon_{t+s}^{\frac{\alpha}{1-\alpha}}} \quad \left[\tilde{P}_t(i) \ \frac{P_t(i)}{P_t(i)} \pi_{t,t+s} - (1+\lambda_{p,t+s})\, MC_{t+s} \ \frac{P_{t+s}}{P_{t+s}} \right] \tilde{Y}_{t+s}(i) \ \frac{A_{t+s} \Upsilon_{t+s}^{\frac{\alpha}{1-\alpha}}}{A_{t+s} \Upsilon_{t+s}^{\frac{\alpha}{1-\alpha}}} = 0$$

$$E_t \sum_{s=0}^{\infty} \xi_p^s \beta^s \lambda_{t+s} \quad \left[\tilde{p}_t(i) \pi_{t,t+s} \ \frac{P_t(i)}{P_{t+s}} - (1+\lambda_{p,t+s})\, s_{t+s} \right] \tilde{y}_{t+s}(i) = 0$$

$$E_t \sum_{s=0}^{\infty} \xi_p^s \beta^s \lambda_{t+s} \quad \left[\tilde{p}_t(i) \tilde{\pi}_{t,t+s} - (1+\lambda_{p,t+s})\, s_{t+s} \right] \tilde{y}_{t+s}(i) = 0 \tag{D.7}$$

where the stationary inflation, $\tilde{\pi}_{t,t+s}$, is given by

$$\pi_{t,t+s} = \prod_{j=0}^{s} \pi_{t+j-1}^{\iota_p} \pi^{1-\iota} = \prod_{j=0}^{s} \left(\frac{\pi_{t+j-1}}{\pi} \right)^{\iota_p} \pi$$

so,

$$\tilde{\pi}_{t,t+s} = \prod_{j=0}^{s} \left(\frac{\pi_{t+j-1}}{\pi} \right)^{\iota_p} \pi \frac{P_t(i)}{P_{t+j}} = \prod_{j=0}^{s} \left(\frac{\pi_{t+j-1}}{\pi} \right)^{\iota_p} \left(\frac{\pi_{t+j}}{\pi} \right)^{-1}$$

Given that, in steady state $\tilde{p} = \tilde{P}/P = 1$, and, therefore, $\tilde{\pi}_{t,t+s} = 1$, expression (D.7), in steady state, becomes

$$\xi_p \beta \lambda \quad \left[\tilde{p} \tilde{\pi} - (1+\lambda_p)\, s \right] \tilde{y} = 0$$

$$1 - (1+\lambda_p)\, s = 0$$

$$s = \frac{1}{1+\lambda_p} \tag{D.8}$$

The aggregate price index given in (B.12), in terms of stationary variables, becomes

$$P_t = \left[\xi_p \left(P_{t-1} \frac{P_t}{P_t} \pi_{t-1}^{\iota_p} \pi^{1-\iota_p} \right)^{-\frac{1}{\lambda_{p,t}}} + (1-\xi_p) \left(\tilde{P}_t \frac{P_t}{P_t} \right)^{-\frac{1}{\lambda_{p,t}}} \right]^{-\lambda_{p,t}}$$

$$P_t = \left[P_t^{-\frac{1}{\lambda_{p,t}}} \xi_p \left(\frac{P_{t-1}}{P_t} \pi_{t-1}^{\iota_p} \pi^{1-\iota_p} \right)^{-\frac{1}{\lambda_{p,t}}} + P_t^{-\frac{1}{\lambda_{p,t}}} (1-\xi_p) \left(\frac{\tilde{P}_t}{P_t} \right)^{-\frac{1}{\lambda_{p,t}}} \right]^{-\lambda_{p,t}}$$

$$1 = \left\{ \xi_p \left[\left(\frac{\pi_{t-1}}{\pi} \right)^{\iota_p} \left(\frac{\pi_t}{\pi} \right)^{-1} \right]^{-\frac{1}{\lambda_{p,t}}} + (1-\xi_p)\, \tilde{p}_t^{-\frac{1}{\lambda_{p,t}}} \right\}^{-\lambda_{p,t}} \tag{D.9}$$

which in steady state reduces to

$$1 = \left\{ \xi_p \left[(1)^{\iota_p} (1)^{-1} \right]^{-\frac{1}{\lambda_{p,t}}} + (1-\xi_p)\, 1^{-\frac{1}{\lambda_{p,t}}} \right\}^{-\lambda_{p,t}}$$

$$1 = (\xi_p + 1 - \xi_p)^{-\lambda_{p,t}} = 1$$

Households

The marginal utility for consumption given in (B.21) is detrended as follows

$$\Lambda_t P_t A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}} = A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}} \frac{b_t}{C_t - hC_{t-1}} - A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}} \beta h E_t \Big\{ \frac{b_{t+1}}{C_{t+1} - hC_t} \Big\}$$

$$\lambda_t = A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}} \frac{b_t}{\frac{A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}}}{A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}}} C_t - h \frac{A_{t-1} \Upsilon_{t-1}^{\frac{\alpha}{1-\alpha}}}{A_{t-1} \Upsilon_{t-1}^{\frac{\alpha}{1-\alpha}}} C_{t-1}} - A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}} \beta h E_t \left\{ \frac{b_{t+1}}{\frac{A_{t+1} \Upsilon_{t+1}^{\frac{\alpha}{1-\alpha}}}{A_{t+1} \Upsilon_{t+1}^{\frac{\alpha}{1-\alpha}}} C_{t+1} - h \frac{A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}}}{A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}}} C_t} \right\}$$

$$\lambda_t = \frac{A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}}}{A_{t-1} \Upsilon_{t-1}^{\frac{\alpha}{1-\alpha}}} \frac{b_t}{\frac{A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}}}{A_{t-1} \Upsilon_{t-1}^{\frac{\alpha}{1-\alpha}}} c_t - hc_{t-1}} - \frac{A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}}}{A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}}} \beta h E_t \left\{ \frac{b_{t+1}}{\frac{A_{t+1} \Upsilon_{t+1}^{\frac{\alpha}{1-\alpha}}}{A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}}} c_{t+1} - hc_t} \right\}$$

Now, note that, the growth rates of the two non stationary shocks, i.e. the IST shock, v_t and the labor-augmenting technology shock, z_t, can be expressed as follows, respectively

$$v_t = \Delta \log \Upsilon_t = \log \Upsilon_t - \log \Upsilon_{t-1} = \log \frac{\Upsilon_t}{\Upsilon_{t-1}}$$

$$z_t = \Delta \log A_t = \log A_t - \log A_{t-1} = \log \frac{A_t}{A_{t-1}}$$

implying that

$$\left(\frac{\Upsilon_t}{\Upsilon_{t-1}} \right)^{\frac{\alpha}{1-\alpha}} = e^{\frac{\alpha}{1-\alpha} v_t}$$

$$\frac{A_t}{A_{t-1}} = e^{z_t}$$

Therefore, the detrended marginal utility of consumption becomes

$$\lambda_t = \frac{e^{z_t + \frac{\alpha}{1-\alpha} v_t} b_t}{e^{z_t + \frac{\alpha}{1-\alpha} v_t} c_t - hc_{t-1}} - \beta h E_t \left\{ \frac{b_{t+1}}{e^{z_{t+1} + \frac{\alpha}{1-\alpha} v_{t+1}} c_{t+1} - hc_t} \right\} \tag{D.10}$$

Next, note that the steady state expressions of the intertemporal preference shock, b_t, and of the growth rates of the IST shock, v_t, and the labor-augmenting technology shock, z_t, are, respectively

$$\log b_t = \rho_b \log b_{t-1} + \varepsilon_{b,t} \qquad \Rightarrow \qquad b = e^{\varepsilon_b} = e^0 = 1,$$

$$v_t = (1 - \rho_v)\gamma_v + \rho_v v_{t-1} + \varepsilon_{v,t} \qquad \Rightarrow \qquad e^v = e^{\gamma_v},$$

$$z_t = (1-\rho_z)\,\gamma_z + \rho_z z_{t-1} + \varepsilon_{z,t} \qquad \Rightarrow \qquad e^z = e^{\gamma_z}$$

Therefore, equation (D.10) in steady state becomes

$$\lambda = \frac{e^{\gamma_z + \frac{\alpha}{1-\alpha}\gamma_v}}{e^{\gamma_z + \frac{\alpha}{1-\alpha}\gamma_v} c - hc} - \beta h \frac{1}{e^{\gamma_z + \frac{\alpha}{1-\alpha}\gamma_v} c - hc}$$

$$\lambda c = \frac{e^{\gamma_z + \frac{\alpha}{1-\alpha}\gamma_v} - \beta h}{e^{\gamma_z + \frac{\alpha}{1-\alpha}\gamma_v} - h} \tag{D.11}$$

The stationary Euler equation (B.22) is calculated as follows

$$\Lambda_t P_t A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}} = P_t A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}} \beta R_t E_t \Big\{ \frac{P_{t+1} A_{t+1} \Upsilon_{t+1}^{\frac{\alpha}{1-\alpha}}}{P_{t+1} A_{t+1} \Upsilon_{t+1}^{\frac{\alpha}{1-\alpha}}} \Lambda_{t+1} \Big\}$$

$$\lambda_t = \beta R_t E_t \Big\{ \lambda_{t+1} e^{-\left(z_{t+1} + \frac{\alpha}{1-\alpha} v_{t+1}\right)} \pi_{t+1}^{-1} \Big\} \tag{D.12}$$

which in steady state becomes

$$\beta = \frac{e^{\gamma_z + \frac{\alpha}{1-\alpha}\gamma_v}}{R} \tag{D.13}$$

The optimal capital supply condition (B.25), given that $P_t^I = P_t \Upsilon_t^{-1}$, is detrended as follows

$$\Lambda_t P_t \Upsilon_t^{-1} A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}} = A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}} \Phi_t \mu_t \Bigg[1 - S\left(\frac{I_t}{I_{t-1}} \frac{A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}+1}}{A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}+1}} \frac{A_{t-1} \Upsilon_{t-1}^{\frac{\alpha}{1-\alpha}+1}}{A_{t-1} \Upsilon_{t-1}^{\frac{\alpha}{1-\alpha}+1}} \right) +$$

$$- \frac{I_t}{I_{t-1}} \frac{A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}+1}}{A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}+1}} \frac{A_{t-1} \Upsilon_{t-1}^{\frac{\alpha}{1-\alpha}+1}}{A_{t-1} \Upsilon_{t-1}^{\frac{\alpha}{1-\alpha}+1}} S' \left(\frac{I_t}{I_{t-1}} \frac{A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}+1}}{A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}+1}} \frac{A_{t-1} \Upsilon_{t-1}^{\frac{\alpha}{1-\alpha}+1}}{A_{t-1} \Upsilon_{t-1}^{\frac{\alpha}{1-\alpha}+1}} \right) \Bigg] +$$

$$+ A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}} \beta E_t \left\{ \frac{A_{t+1} \Upsilon_{t+1}^{\frac{\alpha}{1-\alpha}+1}}{A_{t+1} \Upsilon_{t+1}^{\frac{\alpha}{1-\alpha}+1}} \Phi_{t+1} \mu_{t+1} \left(\frac{I_{t+1}}{I_t} \frac{A_{t+1} \Upsilon_{t+1}^{\frac{\alpha}{1-\alpha}+1}}{A_{t+1} \Upsilon_{t+1}^{\frac{\alpha}{1-\alpha}+1}} \frac{A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}+1}}{A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}+1}} \right)^2 \times \right.$$

$$\left. \times S' \left(\frac{I_{t+1}}{I_t} \frac{A_{t+1} \Upsilon_{t+1}^{\frac{\alpha}{1-\alpha}+1}}{A_{t+1} \Upsilon_{t+1}^{\frac{\alpha}{1-\alpha}+1}} \frac{A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}+1}}{A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}+1}} \right) \right\}$$

$$\lambda_t = \phi_t \mu_t \left[1 - S\left(\frac{i_t}{i_{t-1}} e^{z_t + \left(\frac{\alpha}{1-\alpha}+1\right) v_t} \right) - \frac{i_t}{i_{t-1}} e^{z_t + \left(\frac{\alpha}{1-\alpha}+1\right) v_t} S' \left(\frac{i_t}{i_{t-1}} e^{z_t + \left(\frac{\alpha}{1-\alpha}+1\right) v_t} \right) \right] +$$

$$+ \beta E_t \left\{ \phi_{t+1} \mu_{t+1} e^{-\left(z_{t+1} + \left(\frac{\alpha}{1-\alpha}+1\right) v_{t+1}\right)} \left(\frac{i_{t+1}}{i_t} e^{z_{t+1} + \left(\frac{\alpha}{1-\alpha}+1\right) v_{t+1}} \right)^2 S' \left(\frac{i_{t+1}}{i_t} e^{z_{t+1} + \left(\frac{\alpha}{1-\alpha}+1\right) v_{t+1}} \right) \right\} \tag{D.14}$$

Given that, in steady state $S = S' = 0$, $S'' > 0$, and that the MEI shock equals unity, i.e. $\mu = 1$, equation (D.14), in steady state, becomes

$$\lambda = \phi\Bigg[1 - 0 - 0\Bigg] + \beta * 0$$

$$\lambda = \phi \tag{D.15}$$

Before detrending the optimal condition for maintenance goods, note that, the zero profit condition of the maintenance goods producing sector implies that

$$P_t^m M_t = P_t Y_t^m$$

$$Y_t^m = \frac{P_t^m}{P_t} M_t$$

where $Y_t^m = \tilde{M}_t$ represents the real maintenance in consumption units. Therefore, detrending the production technology for maintenance I obtain

$$M_t \frac{A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}}}{A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}}} = d_t Y_t^m \frac{A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}}}{A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}}}\left[1 - f\left(\frac{Y_t^m}{Y_{t-1}^m} \frac{A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}}}{A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}}}\right)\right]$$

$$m_t = d_t \tilde{m}_t \left[1 - f\left(\frac{\tilde{m}_t}{\tilde{m}_{t-1}} e^{z_t + \frac{\alpha}{1-\alpha} v_t}\right)\right] \tag{D.16}$$

which, recalling that in steady state $f = f' = 0$ and $d = 1$, gives the following identity in steady state

$$m = \tilde{m}$$

Hence, I calculate the detrended counterpart of the maintenance optimal condition, (B.27), as follows

$$\Lambda_t P_t \frac{A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}}}{A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}}} = \left[\sigma \frac{A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}+1}}{A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}+1}} \Phi_t \Upsilon_{t-1}^{-\sigma} \left(\frac{M_t}{\bar{K}_{t-1}} \frac{A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}}}{A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}}} \frac{A_{t-1} \Upsilon_{t-1}^{\frac{\alpha}{1-\alpha}+1}}{A_{t-1} \Upsilon_{t-1}^{\frac{\alpha}{1-\alpha}+1}}\right)^{-1} \left(D_t \frac{\Upsilon_t^\sigma}{\Upsilon_t^\sigma} - \Upsilon_t^\sigma \bar{\delta}\right) - \Gamma_t \frac{A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}}}{A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}}}\right] \times$$

$$\times\, d_t \left[1 - f\left(\frac{\tilde{M}_t}{\tilde{M}_{t-1}} \frac{A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}}}{A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}}} \frac{A_{t-1} \Upsilon_{t-1}^{\frac{\alpha}{1-\alpha}}}{A_{t-1} \Upsilon_{t-1}^{\frac{\alpha}{1-\alpha}}}\right) - \frac{\tilde{M}_t}{\tilde{M}_{t-1}} \frac{A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}}}{A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}}} \frac{A_{t-1} \Upsilon_{t-1}^{\frac{\alpha}{1-\alpha}}}{A_{t-1} \Upsilon_{t-1}^{\frac{\alpha}{1-\alpha}}} f'\left(\frac{\tilde{M}_t}{\tilde{M}_{t-1}} \frac{A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}}}{A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}}} \frac{A_{t-1} \Upsilon_{t-1}^{\frac{\alpha}{1-\alpha}}}{A_{t-1} \Upsilon_{t-1}^{\frac{\alpha}{1-\alpha}}}\right)\right] +$$

$$+\, \beta E_t \Bigg\{ \Big[\sigma \frac{A_{t+1} \Upsilon_{t+1}^{\frac{\alpha}{1-\alpha}+1}}{A_{t+1} \Upsilon_{t+1}^{\frac{\alpha}{1-\alpha}+1}} \Phi_{t+1} \Upsilon_t^{-\sigma} \left(\frac{M_{t+1}}{\bar{K}_t} \frac{A_{t+1} \Upsilon_{t+1}^{\frac{\alpha}{1-\alpha}}}{A_{t+1} \Upsilon_{t+1}^{\frac{\alpha}{1-\alpha}}} \frac{A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}+1}}{A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}+1}}\right)^{-1} \left(\frac{\Upsilon_{t+1}^\sigma}{\Upsilon_{t+1}^\sigma} D_{t+1} - \Upsilon_{t+1}^\sigma \bar{\delta}\right) +$$

$$-\, \Gamma_{t+1} \frac{A_{t+1} \Upsilon_{t+1}^{\frac{\alpha}{1-\alpha}}}{A_{t+1} \Upsilon_{t+1}^{\frac{\alpha}{1-\alpha}}}\Big] \times d_{t+1} \left(\frac{\tilde{M}_{t+1}}{\tilde{M}_t} \frac{A_{t+1} \Upsilon_{t+1}^{\frac{\alpha}{1-\alpha}}}{A_{t+1} \Upsilon_{t+1}^{\frac{\alpha}{1-\alpha}}} \frac{A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}}}{A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}}}\right)^2 f'\left(\frac{\tilde{M}_{t+1}}{\tilde{M}_t} \frac{A_{t+1} \Upsilon_{t+1}^{\frac{\alpha}{1-\alpha}}}{A_{t+1} \Upsilon_{t+1}^{\frac{\alpha}{1-\alpha}}} \frac{A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}}}{A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}}}\right)\Bigg\}$$

which becomes

$$\lambda_t = \left[\sigma\phi_t\left(\frac{m_t}{\bar{k}_{t-1}}e^{z_t+\left(\frac{\alpha}{1-\alpha}+1\right)v_t}\right)^{-1}\left(\delta_t-\bar{\delta}\right)e^{\sigma v_t}-\varsigma_t\right]\times$$
$$\times\, d_t\left[1-f\left(\frac{\tilde{m}_t}{\tilde{m}_{t-1}}e^{z_t+\frac{\alpha}{1-\alpha}v_t}\right)-\frac{\tilde{m}_t}{\tilde{m}_{t-1}}e^{z_t+\frac{\alpha}{1-\alpha}v_t}f'\left(\frac{\tilde{m}_t}{\tilde{m}_{t-1}}e^{z_t+\frac{\alpha}{1-\alpha}v_t}\right)\right]+ \tag{D.17}$$
$$+\,\beta E_t\Bigg\{\left[\sigma\phi_{t+1}\left(\frac{m_{t+1}}{\bar{k}_t}e^{z_{t+1}+\left(\frac{\alpha}{1-\alpha}+1\right)v_{t+1}}\right)^{-1}\left(\delta_{t+1}-\bar{\delta}\right)e^{\sigma v_{t+1}}-\varsigma_{t+1}\right]d_{t+1}e^{-\left(z_{t+1}+\frac{\alpha}{1-\alpha}v_{t+1}\right)}\times$$
$$\times\left(\frac{\tilde{m}_{t+1}}{\tilde{m}_t}e^{z_{t+1}+\frac{\alpha}{1-\alpha}v_{t+1}}\right)^2 f'\left(\frac{\tilde{m}_{t+1}}{\tilde{m}_t}e^{z_{t+1}+\frac{\alpha}{1-\alpha}v_{t+1}}\right)\Bigg\}$$

The above expression in steady state becomes

$$\lambda = \sigma\phi e^{\sigma\gamma_v}\left(\delta-\bar{\delta}\right)\left[\frac{\tilde{m}}{\bar{k}}e^{\gamma_z+\left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v}\right]^{-1}-\varsigma \tag{D.18}$$

Next, I detrend the optimal capital utilization rate condition, (B.28), that is

$$\Lambda_t R_t^k \frac{P_t A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}}}{P_t A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}}}\frac{P_t\Upsilon_t^{-1}}{P_t\Upsilon_t^{-1}} = \Lambda_t P_t \frac{A_t\Upsilon_t^{\frac{\alpha}{1-\alpha}}}{A_t\Upsilon_t^{\frac{\alpha}{1-\alpha}}}\Upsilon_t^{-1}a'\left(u_t\right)+$$
$$+\,\eta\Phi_t\frac{A_t\Upsilon_t^{\frac{\alpha}{1-\alpha}+1}}{A_t\Upsilon_t^{\frac{\alpha}{1-\alpha}+1}}\Upsilon_{t-1}^{-\sigma}u_t^{-1}\left(D_t\frac{\Upsilon_t^\sigma}{\Upsilon_t^\sigma}-\Upsilon_t^\sigma\bar{\delta}\right)-\tau\Gamma_t\frac{A_t\Upsilon_t^{\frac{\alpha}{1-\alpha}}}{A_t\Upsilon_t^{\frac{\alpha}{1-\alpha}}}\Upsilon_t^{-1}$$
$$\lambda_t\rho_t = \lambda_t a'\left(u_t\right)+\eta\phi_t\left(\delta_t-\bar{\delta}\right)e^{\sigma v_t}u_t^{-1}-\tau\varsigma_t \tag{D.19}$$

Given the steady state assumptions $u_t = u = 1$, $a(1) = 0$, and $a'(1) > 0$, the above expression becomes

$$\lambda\rho = \lambda a'\left(1\right)+\eta\phi\left(\delta-\bar{\delta}\right)e^{\sigma\gamma_v}-\tau\varsigma$$
$$\rho = a'\left(1\right)+\eta\frac{\phi}{\lambda}\left(\delta-\bar{\delta}\right)e^{\sigma\gamma_v}-\tau\frac{\varsigma}{\lambda} \tag{D.20}$$

The detrended optimal demand for capital stock, (B.29), is derived as follows

$$\Phi_t\frac{A_t\Upsilon_t^{\frac{\alpha}{1-\alpha}+1}}{A_t\Upsilon_t^{\frac{\alpha}{1-\alpha}+1}} = \beta\Lambda_{t+1}\frac{P_{t+1}A_{t+1}\Upsilon_{t+1}^{\frac{\alpha}{1-\alpha}}}{P_{t+1}A_{t+1}\Upsilon_{t+1}^{\frac{\alpha}{1-\alpha}}}\left[R_{t+1}^k u_{t+1}\frac{P_{t+1}\Upsilon_{t+1}^{-1}}{P_{t+1}\Upsilon_{t+1}^{-1}}-P_{t+1}\Upsilon_{t+1}^{-1}a\left(u_{t+1}\right)\right]+$$
$$+\,\beta\Phi_{t+1}\frac{A_{t+1}\Upsilon_{t+1}^{\frac{\alpha}{1-\alpha}+1}}{A_{t+1}\Upsilon_{t+1}^{\frac{\alpha}{1-\alpha}+1}}\left(1-\Upsilon_t^{-\sigma}D_{t+1}\frac{\Upsilon_{t+1}^\sigma}{\Upsilon_{t+1}^\sigma}\right)+\beta\tau\Gamma_{t+1}\frac{A_{t+1}\Upsilon_{t+1}^{\frac{\alpha}{1-\alpha}}}{A_{t+1}\Upsilon_{t+1}^{\frac{\alpha}{1-\alpha}}}\Upsilon_{t+1}^{-1}u_{t+1}+$$

$$- \beta\sigma\Phi_{t+1}\frac{A_{t+1}\Upsilon_{t+1}^{\frac{\alpha}{1-\alpha}+1}}{A_{t+1}\Upsilon_{t+1}^{\frac{\alpha}{1-\alpha}+1}}\Upsilon_t^{-\sigma}\left(D_{t+1}\frac{\Upsilon_{t+1}^{\sigma}}{\Upsilon_{t+1}^{\sigma}} - \Upsilon_{t+1}^{\sigma}\bar{\delta}\right)$$

$$\phi_t = \beta E_t \Bigg\{ \lambda_{t+1}\left[\rho_{t+1}u_{t+1} - a\left(u_{t+1}\right)\right]e^{-\left(z_{t+1}+\left(\frac{\alpha}{1-\alpha}+1\right)v_{t+1}\right)} + \tau\varsigma_{t+1}u_{t+1}e^{-\left(z_{t+1}+\left(\frac{\alpha}{1-\alpha}+1\right)v_{t+1}\right)} +$$

$$+ \phi_{t+1}\left(1-\delta_{t+1}e^{\sigma v_{t+1}}\right)e^{-\left(z_{t+1}+\left(\frac{\alpha}{1-\alpha}+1\right)v_{t+1}\right)} - \sigma\phi_{t+1}\left(\delta_{t+1}-\bar{\delta}\right)e^{\sigma v_{t+1}}e^{-\left(z_{t+1}+\left(\frac{\alpha}{1-\alpha}+1\right)v_{t+1}\right)} \Bigg\} \tag{D.21}$$

which, using the expression (D.18), in steady state, becomes

$$\phi\left[e^{\gamma_z+\left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v} - \beta\left(1-\delta e^{\sigma\gamma_v}\right) + \beta\sigma e^{\sigma\gamma_v}\left(\delta-\bar{\delta}\right)\right] = \beta\lambda\rho +$$

$$+ \beta\tau\left[\sigma\phi e^{\sigma\gamma_v}\left(\delta-\bar{\delta}\right)\left(\frac{\tilde{m}}{\tilde{k}}e^{\gamma_z+\left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v}\right)^{-1} - \lambda\right]$$

$$\phi = \frac{\lambda\beta\left(\rho-\tau\right)}{e^{\gamma_z+\left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v} - \beta\left(1-\delta e^{\sigma\gamma_v}\right) + \beta\sigma\left(\delta-\bar{\delta}\right)e^{\sigma\gamma_v}\left[1-\tau\left(\frac{\tilde{m}}{\tilde{k}}e^{\gamma_z+\left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v}\right)^{-1}\right]} \tag{D.22}$$

Recalling the optimal wage setting condition given in (B.30), I detrend it as follows

$$E_t\sum_{s=0}^{\infty}\xi_w^s\beta^s\Lambda_{t+s}\frac{P_{t+s}A_{t+s}\Upsilon_{t+s}^{\frac{\alpha}{1-\alpha}}}{P_{t+s}A_{t+s}\Upsilon_{t+s}^{\frac{\alpha}{1-\alpha}}}\tilde{L}_{t+s}(j)\Bigg[\pi_{t,t+s}^{w}\tilde{W}_t(j)\frac{P_tA_t\Upsilon_t^{\frac{\alpha}{1-\alpha}}}{P_tA_t\Upsilon_t^{\frac{\alpha}{1-\alpha}}} +$$

$$- b_{t+s}\varphi\left(1+\lambda_{w,t+s}\right)\frac{\tilde{L}_{t+s}(j)^{\nu}}{\Lambda_{t+s}}\frac{P_{t+s}A_{t+s}\Upsilon_{t+s}^{\frac{\alpha}{1-\alpha}}}{P_{t+s}A_{t+s}\Upsilon_{t+s}^{\frac{\alpha}{1-\alpha}}}\Bigg] = 0$$

$$E_t\sum_{s=0}^{\infty}\xi_w^s\beta^s\lambda_{t+s}\tilde{L}_{t+s}(j)\left[\tilde{w}_t(j)\pi_{t,t+s}^{w}\frac{P_tA_t\Upsilon_t^{\frac{\alpha}{1-\alpha}}}{P_{t+s}A_{t+s}\Upsilon_{t+s}^{\frac{\alpha}{1-\alpha}}} - b_{t+s}\varphi\left(1+\lambda_{w,t+s}\right)\frac{\tilde{L}_{t+s}(j)^{\nu}}{\lambda_{t+s}}\right] = 0$$

$$E_t\sum_{s=0}^{\infty}\xi_w^s\beta^s\lambda_{t+s}\tilde{L}_{t+s}(j)\left[\tilde{w}_t(j)\tilde{\pi}_{t,t+s}^{w} - b_{t+s}\varphi\left(1+\lambda_{w,t+s}\right)\frac{\tilde{L}_{t+s}(j)^{\nu}}{\lambda_{t+s}}\right] = 0 \tag{D.23}$$

where $\tilde{\pi}_{t,t+s}^{w}$ is defined as follows

$$\tilde{\pi}_{t,t+s}^{w} = \prod_{k=0}^{s}\left[\left(\pi_{t+k-1}e^{z_{t+k-1}+\frac{\alpha}{1-\alpha}v_{t+k-1}}\right)^{\iota_w}\left(\pi e^{\gamma_z+\frac{\alpha}{1-\alpha}\gamma_v}\right)^{1-\iota_w}\frac{P_tA_t\Upsilon_t^{\frac{\alpha}{1-\alpha}}}{P_{t+k}A_{t+k}\Upsilon_{t+k}^{\frac{\alpha}{1-\alpha}}}\right]$$

$$= \prod_{k=0}^{s} \left[\left(\frac{\pi_{t+k-1} e^{z_{t+k-1} + \frac{\alpha}{1-\alpha} v_{t+k-1}}}{\pi e^{\gamma_z + \frac{\alpha}{1-\alpha}\gamma_v}} \right)^{\iota_w} \left(\frac{\pi_{t+k} e^{z_{t+k} + \frac{\alpha}{1-\alpha} v_{t+k}}}{\pi e^{\gamma_z + \frac{\alpha}{1-\alpha}\gamma_v}} \right)^{-1} \right]$$

In steady state, expression (D.23) becomes

$$\lambda \tilde{L}(j) \left[\tilde{w}(j) - \varphi \left(1 + \lambda_w\right) \frac{\tilde{L}(j)^{\nu}}{\lambda} \right] = 0$$

$$\tilde{L}(j)^{\nu} = \frac{\lambda}{\varphi} \frac{\tilde{w}(j)}{1 + \lambda_w} \tag{D.24}$$

Recall that the optimal demand for labor is given by

$$L_{t+s}(j) = \left[\frac{W_t(j)}{W_{t+s}} \pi^{w}_{t,t+s} \right]^{-\frac{1+\lambda_{w,t+s}}{\lambda_{w,t+s}}} L_{t+s}$$

Detrending it is obtained

$$L_{t+s}(j) = \left[\frac{W_t(j)}{W_{t+s}} \; \frac{P_{t+s} A_{t+s} \Upsilon_{t+s}^{\frac{\alpha}{1-\alpha}}}{P_{t+s} A_{t+s} \Upsilon_{t+s}^{\frac{\alpha}{1-\alpha}}} \; \frac{P_t A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}}}{P_t A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}}} \; \pi^{w}_{t,t+s} \right]^{-\frac{1+\lambda_{w,t+s}}{\lambda_{w,t+s}}} L_{t+s}$$

$$L_{t+s}(j) = \left[\frac{w_t(j)}{w_{t+s}} \tilde{\pi}^{w}_{t,t+s} \right]^{-\frac{1+\lambda_{w,t+s}}{\lambda_{w,t+s}}} L_{t+s} \tag{D.25}$$

The aggregate wage index (B.31) in the stationary equilibrium is derived as follows

$$\frac{W_t}{P_t A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}}} = \left\{ \xi_w \left[\frac{W_{t-1}}{P_{t-1} A_{t-1} \Upsilon_{t-1}^{\frac{\alpha}{1-\alpha}}} \frac{P_{t-1} A_{t-1} \Upsilon_{t-1}^{\frac{\alpha}{1-\alpha}}}{P_t A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}}} \left(\pi_{t-1} e^{z_{t-1} + \frac{\alpha}{1-\alpha} v_{t-1}} \right)^{\iota_w} \left(\pi e^{\gamma_z + \frac{\alpha}{1-\alpha}\gamma_v} \right)^{1-\iota_w} \right]^{-\frac{1}{\lambda_{w,t}}} + \right.$$

$$\left. + \left(1 - \xi_w\right) \left[\frac{\tilde{W}_t(j)}{P_t A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}}} \right]^{-\frac{1}{\lambda_{w,t}}} \right\}^{-\lambda_{w,t}}$$

$$w_t = \left\{ \xi_w \left[w_{t-1} \left(\frac{\pi_{t-1} e^{z_{t-1} + \frac{\alpha}{1-\alpha} v_{t-1}}}{\pi e^{\gamma_z + \frac{\alpha}{1-\alpha}\gamma_v}} \right)^{\iota_w} \left(\frac{\pi_t e^{z_t + \frac{\alpha}{1-\alpha} v_t}}{\pi e^{\gamma_z + \frac{\alpha}{1-\alpha}\gamma_v}} \right)^{-1} \right]^{-\frac{1}{\lambda_{w,t}}} + \left(1 - \xi_w\right) \tilde{w}_t(j)^{-\frac{1}{\lambda_{w,t}}} \right\}^{-\lambda_{w,t}} \tag{D.26}$$

The definition of capital utilization, $K_t = u_t \bar{K}_{t-1}$, in stationary equilibrium is given by

$$K_t \frac{A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}+1}}{A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}+1}} = u_t \frac{A_{t-1} \Upsilon_{t-1}^{\frac{\alpha}{1-\alpha}+1}}{A_{t-1} \Upsilon_{t-1}^{\frac{\alpha}{1-\alpha}+1}} \bar{K}_{t-1}$$

$$k_t = u_t \bar{k}_{t-1} e^{-\left(z_t + \left(\frac{\alpha}{1-\alpha}+1\right) v_t\right)} \tag{D.27}$$

which in steady state becomes

$$k = \bar{k} e^{-\left(\gamma_z + \left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v\right)} \tag{D.28}$$

The definition of the capital depreciation rate is given by

$$D_t = \zeta u_t^\eta \left(\frac{M_t}{\bar{K}_{t-1}}\right)^{-\sigma} + \Upsilon_t^\sigma \bar{\delta}$$

which in the stationary equilibrium model becomes

$$D_t \frac{\Upsilon_t^\sigma}{\Upsilon_t^\sigma} = \zeta u_t^\eta \left(\frac{M_t}{\bar{K}_{t-1}} \frac{A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}}}{A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}}} \frac{A_{t-1} \Upsilon_{t-1}^{\frac{\alpha}{1-\alpha}+1}}{A_{t-1} \Upsilon_{t-1}^{\frac{\alpha}{1-\alpha}+1}}\right)^{-\sigma} + \Upsilon_t^\sigma \bar{\delta}$$

$$\delta_t = \zeta u_t^\eta \left(\frac{m_t}{\bar{k}_{t-1}} e^{z_t + \left(\frac{\alpha}{1-\alpha}+1\right) v_t}\right)^{-\sigma} + \bar{\delta} \tag{D.29}$$

and in steady state, recalling that $m = \tilde{m}$, is given by

$$\delta = \zeta \left(\frac{\tilde{m}}{\bar{k}} e^{\gamma_z + \left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v}\right)^{-\sigma} + \bar{\delta} \tag{D.30}$$

The definition of maintenance costs is given by

$$M_t = \tau \Upsilon_t^{-1} u_t \bar{K}_{t-1} + A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}} \bar{M}$$

In the stationary equilibrium it becomes

$$M_t \frac{A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}}}{A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}}} = \tau \Upsilon_t^{-1} u_t \bar{K}_{t-1} \frac{A_{t-1} \Upsilon_{t-1}^{\frac{\alpha}{1-\alpha}+1}}{A_{t-1} \Upsilon_{t-1}^{\frac{\alpha}{1-\alpha}+1}} + A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}} \bar{M}$$

$$m_t = \tau u_t \bar{k}_{t-1} e^{-\left(z_t + \left(\frac{\alpha}{1-\alpha}+1\right) v_t\right)} + \bar{M} \tag{D.31}$$

while in steady state it is given by

$$\tilde{m} = \tau \bar{k} e^{-\left(\gamma_z + \left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v\right)} + \bar{M} \tag{D.32}$$

The law of motion of capital, substituting for the technology to produce new capital goods, $i_t^k = \mu_t \left[1 - S\left(I_t / I_{t-1}\right)\right] I_t$ is given by

$$\bar{K}_t = \left(1 - \Upsilon_{t-1}^{-\sigma} D_t\right) \bar{K}_{t-1} + \mu_t \left[1 - S\left(\frac{I_t}{I_{t-1}}\right)\right] I_t$$

which is detrended as follows

$$\bar{K}_t \frac{A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}+1}}{A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}+1}} = \left(1 - \Upsilon_{t-1}^{-\sigma} D_t \frac{\Upsilon_t^{\sigma}}{\Upsilon_t^{\sigma}}\right) \bar{K}_{t-1} \frac{A_{t-1} \Upsilon_{t-1}^{\frac{\alpha}{1-\alpha}+1}}{A_{t-1} \Upsilon_{t-1}^{\frac{\alpha}{1-\alpha}+1}} +$$

$$+ \mu_t \left[1 - S\left(\frac{I_t}{I_{t-1}} \frac{A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}+1}}{A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}+1}} \frac{A_{t-1} \Upsilon_{t-1}^{\frac{\alpha}{1-\alpha}+1}}{A_{t-1} \Upsilon_{t-1}^{\frac{\alpha}{1-\alpha}+1}}\right)\right] I_t \frac{A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}+1}}{A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}+1}}$$

$$\bar{k}_t = \left(1 - \delta_t e^{\sigma v_t}\right) \bar{k}_{t-1} e^{-\left(z_t + \left(\frac{\alpha}{1-\alpha}+1\right) v_t\right)} + \mu_t \left[1 - S\left(\frac{i_t}{i_{t-1}} e^{z_t + \left(\frac{\alpha}{1-\alpha}+1\right) v_t}\right)\right] i_t \tag{D.33}$$

and in steady state it becomes

$$\bar{k} = \left(1 - \delta e^{\sigma \gamma_v}\right) \bar{k} e^{-\left(\gamma_z + \left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v\right)} + [1-0]\, i$$

$$\bar{k} = \frac{i}{1 - \left(1 - \delta e^{\sigma \gamma_v}\right) e^{-\left(\gamma_z + \left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v\right)}} \tag{D.34}$$

The aggregate resource constraint is defined as follows

$$C_t + \Upsilon_t^{-1} I_t + a(u_t)\, \Upsilon_t^{-1} \bar{K}_{t-1} + \tilde{M}_t = (1/g_t)\, Y_t$$

where $\tilde{I}_t = \Upsilon_t^{-1} I_t = (P_t^I/P_t) I_t$, and $\tilde{M}_t = (P_t^m/P_t) M_t$ are the real investment and real maintenance in consumption units, respectively, and $\bar{K}_{t-1}$ is multiplied by Υ_t^{-1} in order to ensure the balanced growth path. Detrending the above expression I obtain

$$C_t \frac{A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}}}{A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}}} + \Upsilon_t^{-1} I_t \frac{A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}}}{A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}}} + a(u_t)\, \Upsilon_t^{-1} \bar{K}_{t-1} \frac{A_{t-1} \Upsilon_{t-1}^{\frac{\alpha}{1-\alpha}+1}}{A_{t-1} \Upsilon_{t-1}^{\frac{\alpha}{1-\alpha}+1}} + \tilde{M}_t \frac{A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}}}{A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}}} = (1/g_t)\, Y_t \frac{A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}}}{A_t \Upsilon_t^{\frac{\alpha}{1-\alpha}}}$$

$$c_t + i_t + a(u_t)\, \bar{k}_{t-1} e^{-\left(z_t + \left(\frac{\alpha}{1-\alpha}+1\right) v_t\right)} + \tilde{m}_t = (1/g_t)\, y_t \tag{D.35}$$

which gives the following expression in steady state

$$c + i + \tilde{m} = (1/g)\, y \tag{D.36}$$

The actual GDP, X_t is given by

$$X_t = (1 - 1/g_t)\, Y_t + C_t + \Upsilon_t^{-1} I_t + \tilde{M}_t$$

which detrended becomes

$$x_t = (1 - 1/g_t)\, y_t + c_t + i_t + \tilde{m}_t \tag{D.37}$$

and in steady state it is

$$x = (1 - 1/g)\, y + c + i + \tilde{m} \tag{D.38}$$

The stationary expression for the monetary policy rule (2.14) is

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\rho_R} \left[\left(\frac{\pi_t}{\pi}\right)^{\phi_\pi} \left(\frac{x_t}{x_t^\star}\right)^{\phi_X}\right]^{1-\rho_R} \left[\frac{x_t/x_{t-1}}{x_t^\star/x_{t-1}^\star}\right]^{\phi_{dX}} \varepsilon_{mp,t} \tag{D.39}$$

E APPENDIX: Linear rational expectations model

In the present section I compute the log-linear approximation of the equilibrium model equations around the non-stochastic steady state. The variables denoted by a hat represent the log-linear deviations form their respective steady state values, that is, given a generic variable H, $\hat{H}_t = \log H_t - \log H$. Moreover, following Justiniano et al. (2011), I apply the following rules of the deviations from steady states of the shocks

$$
\begin{array}{ll}
\hat{\lambda}_{p,t+s} = \ln(1+\lambda_{p,t+s}) - \ln(1+\lambda_p) & \hat{\lambda}_{w,t+s} = \ln(1+\lambda_{w,t+s}) - \ln(1+\lambda_w) \\
\hat{d}_t = d_t - 1 & \hat{g}_t = g_t - g \\
\hat{b}_t = b_t - 1 & \hat{\mu}_t = \mu_t - 1 \\
\hat{z}_t = z_t - \gamma_z & \hat{v}_t = v_t - \gamma_v
\end{array}
$$

Moreover, the expectations of the stationary shocks are given by

$$
\begin{array}{ll}
\ln E_t\{b_{t+1}\} = \rho_b \ln b_t & \ln E_t\{d_{t+1}\} = \rho_d \ln d_t \\
E_t\{z_{t+1}\} = \rho_z z_t & \ln E_t\{\mu_{t+1}\} = \rho_\mu \ln \mu_t \\
E_t\{v_{t+1}\} = \rho_v v_t & \ln E_t\{g_{t+1}\} = \rho_g \ln g_t
\end{array}
$$

Recall the detrended aggregate production function of the intermediate goods producers, (D.1), and its steady state expression, (D.2). I log-linearize it as follows

$$\ln[y_t(i)] = \ln\left[k_t(i)^\alpha L_t(i)^{1-\alpha} - F\right]$$

$$y + \hat{y}_t = y + \frac{1}{y}\alpha k^{\alpha-1} L^{1-\alpha}(k_t - k) + \frac{1}{y}(1-\alpha) k^\alpha L^{1-\alpha-1}(L_t - L)$$

$$\hat{y}_t = \frac{y+F}{y}\alpha \hat{k}_t + \frac{y+F}{y}(1-\alpha)\hat{L}_t \tag{E.1}$$

The optimal relative demand for capital given in (D.3), using its steady state expression, (D.4), is log-linearized as follows

$$\ln\left[\frac{k_t}{L_t}\right] = \ln\left[\frac{\alpha}{1-\alpha}\frac{w_t}{\rho_t}\right]$$

$$\ln k_t - \ln L_t = \ln w_t - \ln \rho_t + \ln\frac{\alpha}{1-\alpha}$$

$$\hat{k}_t - \hat{L}_t = \hat{w}_t - \hat{\rho}_t \tag{E.2}$$

which sets that sets that the marginal returns on effective capital and labor must equal, respectively, the return on capital and the real wage.

Given the optimal detrended nominal marginal costs, (D.5), and the respective steady state relation, (D.6), the log-linearization is as follows

$$\ln[s_t] = \ln\left[\frac{1}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}} w_t^{1-\alpha}\rho_t^{\alpha}\right]$$

$$\ln s_t = \ln\frac{1}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}} + (1-\alpha)\ln w_t + \alpha\ln\rho_t$$

$$\hat{s}_t = (1-\alpha)\hat{w}_t + \alpha\hat{\rho}_t \tag{E.3}$$

According to which real marginal cost is given by the sum of the return on capital and real wage weighted by the share of capital and labor in the production function, respectively.

The log-linearization of the detrended optimal price setting, given equations (D.7), and (D.8), is

$$\ln\left[E_t\sum_{s=0}^{\infty}\xi_p^s\beta^s\lambda_{t+s}\ \ \tilde{p}_t(i)\tilde{\pi}_{t,t+s}\tilde{y}_{t+s}(i)\right] = \ln\left[E_t\sum_{s=0}^{\infty}\xi_p^s\beta^s\lambda_{t+s}\ \ (1+\lambda_{p,t+s})\,s_{t+s}\tilde{y}_{t+s}(i)\right]$$

$$E_t\sum_{s=0}^{\infty}\xi_p^s\beta^s\left(\ln\tilde{p}_t + \ln\tilde{\pi}_{t,t+s}\right) = E_t\sum_{s=0}^{\infty}\xi_p^s\beta^s\left[\ln(1+\lambda_{p,t+s}) + \ln s_{t+s}\right]$$

$$\frac{1}{1-\xi_p\beta}\hat{\tilde{p}}_t + E_t\sum_{s=0}^{\infty}\xi_p^s\beta^s\hat{\tilde{\pi}}_{t,t+s} = E_t\sum_{s=0}^{\infty}\xi_p^s\beta^s\left(\hat{\lambda}_{p,t+s} + \hat{s}_{t+s}\right)$$

$$\frac{1}{1-\xi_p\beta}\hat{\tilde{p}}_t = E_t\sum_{s=0}^{\infty}\xi_p^s\beta^s\left(-\hat{\tilde{\pi}}_{t,t+s} + \hat{\lambda}_{p,t+s} + \hat{s}_{t+s}\right)$$

$$= -\hat{\tilde{\pi}}_{t,t} + \hat{\lambda}_{p,t} + \hat{s}_t + E_t\sum_{s=1}^{\infty}\xi_p^s\beta^s\left(-\hat{\tilde{\pi}}_{t,t+s} + \hat{\lambda}_{p,t+s} + \hat{s}_{t+s}\right)$$

$$= 0 + \hat{\lambda}_{p,t} + \hat{s}_t + \xi_p\beta E_t\sum_{s=0}^{\infty}\xi_p^s\beta^s\left(-\hat{\tilde{\pi}}_{t,t+s+1} + \hat{\lambda}_{p,t+s+1} + \hat{s}_{t+s+1}\right)$$

$$= \hat{\lambda}_{p,t} + \hat{s}_t - \xi_p\beta E_t\sum_{s=0}^{\infty}\xi_p^s\beta^s\hat{\tilde{\pi}}_{t,t+1} + \xi_p\beta E_t\sum_{s=0}^{\infty}\xi_p^s\beta^s\left(-\hat{\tilde{\pi}}_{t+1,t+s+1} + \hat{\lambda}_{p,t+s+1} + \hat{s}_{t+s+1}\right)$$

$$= \hat{\lambda}_{p,t} + \hat{s}_t - \frac{\xi_p\beta}{1-\xi_p\beta}E_t\{\hat{\tilde{\pi}}_{t,t+1}\} + \frac{\xi_p\beta}{1-\xi_p\beta}E_t\{\hat{\tilde{p}}_{t+1}\}$$

Therefore,

$$\frac{1}{1-\xi_p\beta}\hat{\tilde{p}}_t = \hat{\lambda}_{p,t} + \hat{s}_t + \frac{\xi_p\beta}{1-\xi_p\beta}E_t\{\hat{\tilde{p}}_{t+1} - \hat{\tilde{\pi}}_{t,t+1}\} \tag{E.4}$$

The log-linear detrended aggregate price index, (D.9) is given by

$$1^{-\frac{1}{\lambda_{p,t}}} = \xi_p\left[\left(\frac{\pi_{t-1}}{\pi}\right)^{\iota_p}\left(\frac{\pi_t}{\pi}\right)^{-1}\right]^{-\frac{1}{\lambda_{p,t}}} + (1-\xi_p)\,\tilde{p}_t^{-\frac{1}{\lambda_{p,t}}}$$

$$\ln 1 = \ln\left\{\xi_p\left[\left(\frac{\pi_{t-1}}{\pi}\right)^{\iota_p}\left(\frac{\pi_t}{\pi}\right)^{-1}\right]^{-\frac{1}{\lambda_{p,t}}} + (1-\xi_p)\,\tilde{p}_t^{-\frac{1}{\lambda_{p,t}}}\right\}$$

$$0 = -\frac{1}{\lambda_{p,t}}\xi_p\left[\left(\frac{\pi}{\pi}\right)^{\iota_p}\left(\frac{\pi}{\pi}\right)^{-1}\right]^{-\frac{1}{\lambda_{p,t}}-1}(\iota_p\hat{\pi}_{t-1} - \hat{\pi}_t) - \frac{1}{\lambda_{p,t}}(1-\xi_p)(\tilde{p}_t - 1)$$

$$0 = \xi_p(\iota_p\hat{\pi}_{t-1} - \hat{\pi}_t) + (1-\xi_p)\,\hat{\tilde{p}}_t \tag{E.5}$$

Combining equation (E.4) with (E.5), both in the current and forward period forms, and using the log-linearized expression for the definition of inflation, i.e. $\hat{\tilde{\pi}}_{t,t+s} = \sum_{j=0}^{s}(\iota_t\hat{\pi}_{t+j-1} - \hat{\pi}_{t+j})$, I obtain the new Phillips curve, as follows

$$\frac{1}{1-\xi_p\beta}\left[-\frac{\xi_p}{1-\xi_p}(\iota_t\hat{\pi}_{t-1} - \hat{\pi}_t)\right] = \hat{\lambda}_{p,t} + \hat{s}_t + \frac{\xi_p\beta}{1-\xi_p\beta}\left[-\frac{\xi_p}{1-\xi_p}(\iota_t\hat{\pi}_t - \hat{\pi}_{t+1})\right] - \frac{\xi_p\beta}{1-\xi_p\beta}(\iota_t\hat{\pi}_t - \hat{\pi}_{t+1})$$

$$\frac{\xi_p + \xi_p\beta\iota_p}{(1-\xi_p\beta)(1-\xi_p)}\hat{\pi}_t = \hat{\lambda}_{p,t} + \hat{s}_t + \frac{\xi_p\iota_p}{(1-\xi_p\beta)(1-\xi_p)}\hat{\pi}_{t-1} + \frac{\xi_p\beta}{(1-\xi_p\beta)(1-\xi_p)}\hat{\pi}_{t+1}$$

$$\hat{\pi}_t = \frac{(1-\xi_p\beta)(1-\xi_p)}{\xi_p(1+\beta\iota_p)}\left(\hat{\lambda}_{p,t} + \hat{s}_t\right) + \frac{\iota_p}{1+\beta\iota_p}\hat{\pi}_{t-1} + \frac{\beta}{1+\beta\iota_p}E_t\{\hat{\pi}_{t+1}\} \tag{E.6}$$

The latter expression represents the New Keynesian Phillips curve for prices, which depends on past and expected inflation as well as on real marginal costs and the price mark-up shock.

Next, I log-linearize the detrended marginal utility of consumption given in (D.10), using its steady state expression (D.11), as follows

$$\ln\lambda_t = \ln\left[\frac{e^{z_t+\frac{\alpha}{1-\alpha}v_t}b_t}{e^{z_t+\frac{\alpha}{1-\alpha}v_t}c_t - hc_{t-1}} - \beta hE_t\left\{\frac{b_{t+1}}{e^{z_{t+1}+\frac{\alpha}{1-\alpha}v_{t+1}}c_{t+1} - hc_t}\right\}\right]$$

$$\hat{\lambda}_t = c\frac{e^{\gamma_z+\frac{\alpha}{1-\alpha}\gamma_v} - h}{e^{\gamma_z+\frac{\alpha}{1-\alpha}\gamma_v} - \beta h}\left\{\left[ce^{\gamma_z+\frac{\alpha}{1-\alpha}\gamma_v}\beta h\left(e^{\gamma_z+\frac{\alpha}{1-\alpha}\gamma_v}c - hc\right)^{-2}\right](z_{t+1} - \gamma_z) + \right.$$

$$+\left[e^{\gamma_z+\frac{\alpha}{1-\alpha}\gamma_v}\left(e^{\gamma_z+\frac{\alpha}{1-\alpha}\gamma_v}c-hc\right)^{-1}-ce^{2\left(\gamma_z+\frac{\alpha}{1-\alpha}\gamma_v\right)}\left(e^{\gamma_z+\frac{\alpha}{1-\alpha}\gamma_v}c-hc\right)^{-2}\right](z_t-\gamma_z)+$$

$$+\frac{\alpha}{1-\alpha}\left[e^{\gamma_z+\frac{\alpha}{1-\alpha}\gamma_v}\left(e^{\gamma_z+\frac{\alpha}{1-\alpha}\gamma_v}c-hc\right)^{-1}-ce^{2\left(\gamma_z+\frac{\alpha}{1-\alpha}\gamma_v\right)}\left(e^{\gamma_z+\frac{\alpha}{1-\alpha}\gamma_v}c-hc\right)^{-2}\right](v_t-\gamma_v)+$$

$$+\frac{\alpha}{1-\alpha}\left[ce^{\gamma_z+\frac{\alpha}{1-\alpha}\gamma_v}\beta h\left(e^{\gamma_z+\frac{\alpha}{1-\alpha}\gamma_v}c-hc\right)^{-2}\right](v_{t+1}-\gamma_v)+$$

$$+e^{\gamma_z+\frac{\alpha}{1-\alpha}\gamma_v}\left(e^{\gamma_z+\frac{\alpha}{1-\alpha}\gamma_v}c-hc\right)^{-1}(b_t-1)-\beta h\left(e^{\gamma_z+\frac{\alpha}{1-\alpha}\gamma_v}c-hc\right)^{-1}(b_{t+1}-1)+$$

$$-e^{2\left(\gamma_z+\frac{\alpha}{1-\alpha}\gamma_v\right)}\left(e^{\gamma_z+\frac{\alpha}{1-\alpha}\gamma_v}c-hc\right)^{-2}(c_t-c)-\beta h^2\left(e^{\gamma_z+\frac{\alpha}{1-\alpha}\gamma_v}c-hc\right)^{-2}(c_t-c)+$$

$$\left.+he^{\gamma_z+\frac{\alpha}{1-\alpha}\gamma_v}\left(e^{\gamma_z+\frac{\alpha}{1-\alpha}\gamma_v}c-hc\right)^{-2}(c_{t-1}-c)+\beta he^{\gamma_z+\frac{\alpha}{1-\alpha}\gamma_v}\left(e^{\gamma_z+\frac{\alpha}{1-\alpha}\gamma_v}c-hc\right)^{-2}(c_{t+1}-c)\right\}$$

$$\hat{\lambda}_t=\frac{1}{e^{\gamma_z+\frac{\alpha}{1-\alpha}\gamma_v}-\beta h}\left\{\beta h\frac{e^{\gamma_z+\frac{\alpha}{1-\alpha}\gamma_v}}{e^{\gamma_z+\frac{\alpha}{1-\alpha}\gamma_v}-h}\hat{z}_{t+1}+\left[e^{\gamma_z+\frac{\alpha}{1-\alpha}\gamma_v}-\frac{e^{2\left(\gamma_z+\frac{\alpha}{1-\alpha}\gamma_v\right)}}{e^{\gamma_z+\frac{\alpha}{1-\alpha}\gamma_v}-h}\right]\hat{z}_t+\right.$$

$$+\frac{\alpha}{1-\alpha}\beta h\frac{e^{\gamma_z+\frac{\alpha}{1-\alpha}\gamma_v}}{e^{\gamma_z+\frac{\alpha}{1-\alpha}\gamma_v}-h}\hat{v}_{t+1}+\frac{\alpha}{1-\alpha}\left[e^{\gamma_z+\frac{\alpha}{1-\alpha}\gamma_v}-\frac{e^{2\left(\gamma_z+\frac{\alpha}{1-\alpha}\gamma_v\right)}}{e^{\gamma_z+\frac{\alpha}{1-\alpha}\gamma_v}-h}\right]\hat{v}_t+$$

$$+e^{\gamma_z+\frac{\alpha}{1-\alpha}\gamma_v}\hat{b}_t-\beta h\hat{b}_{t+1}-\frac{e^{2\left(\gamma_z+\frac{\alpha}{1-\alpha}\gamma_v\right)}}{e^{\gamma_z+\frac{\alpha}{1-\alpha}\gamma_v}-h}\hat{c}_t-\beta h^2\frac{1}{e^{\gamma_z+\frac{\alpha}{1-\alpha}\gamma_v}-h}\hat{c}_t+$$

$$\left.+h\frac{e^{\gamma_z+\frac{\alpha}{1-\alpha}\gamma_v}}{e^{\gamma_z+\frac{\alpha}{1-\alpha}\gamma_v}-h}\hat{c}_{t-1}+\beta h\frac{e^{\gamma_z+\frac{\alpha}{1-\alpha}\gamma_v}}{e^{\gamma_z+\frac{\alpha}{1-\alpha}\gamma_v}-h}\hat{c}_{t+1}\right\}$$

ending up with

$$\begin{aligned}\hat{\lambda}_t=&\frac{1}{\left(e^{\gamma_z+\frac{\alpha}{1-\alpha}\gamma_v}-\beta h\right)\left(e^{\gamma_z+\frac{\alpha}{1-\alpha}\gamma_v}-h\right)}\left\{\left(e^{\gamma_z+\frac{\alpha}{1-\alpha}\gamma_v}-\beta h\rho_b\right)\left(e^{\gamma_z+\frac{\alpha}{1-\alpha}\gamma_v}-h\right)\hat{b}_t+\right.\\&-\left(e^{2\left(\gamma_z+\frac{\alpha}{1-\alpha}\gamma_v\right)}+\beta h^2\right)\hat{c}_t+he^{\gamma_z+\frac{\alpha}{1-\alpha}\gamma_v}\hat{c}_{t-1}+\beta he^{\gamma_z+\frac{\alpha}{1-\alpha}\gamma_v}E_t\{\hat{c}_{t+1}\}+\\&\left.+\left(\beta h\rho_z e^{\gamma_z+\frac{\alpha}{1-\alpha}\gamma_v}-he^{\gamma_z+\frac{\alpha}{1-\alpha}\gamma_v}\right)\hat{z}_t+\frac{\alpha}{1-\alpha}\left(\beta h\rho_v e^{\gamma_z+\frac{\alpha}{1-\alpha}\gamma_v}-he^{\gamma_z+\frac{\alpha}{1-\alpha}\gamma_v}\right)\hat{v}_t\right\}\end{aligned}\tag{E.7}$$

Log-linearizing the detrended Euler equation (D.12) I obtain

$$\ln\lambda_t=\ln\left[\beta R_tE_t\left\{\lambda_{t+1}e^{-\left(z_{t+1}+\frac{\alpha}{1-\alpha}v_{t+1}\right)}\pi_{t+1}^{-1}\right\}\right]$$

$$\ln\lambda_t=\ln\beta+\ln R_t+\ln\lambda_{t+1}-z_{t+1}-\frac{\alpha}{1-\alpha}v_{t+1}-\ln\pi_{t+1}$$

$$\hat{\lambda}_t=\hat{R}_t+E_t\{\hat{\lambda}_{t+1}\}-E_t\{\hat{z}_{t+1}\}-\frac{\alpha}{1-\alpha}E_t\{\hat{v}_{t+1}\}-E_t\{\hat{\pi}_{t+1}\}$$

$$\hat{\lambda}_t = \hat{R}_t - \rho_z \hat{z}_t - \frac{\alpha}{1-\alpha}\rho_v \hat{v}_t + E_t\{\hat{\lambda}_{t+1} - \hat{\pi}_{t+1}\} \tag{E.8}$$

The detrended optimal capital good supply condition, (D.14), making use of (D.15), is log-linearized as follows

$$\begin{aligned}
\ln \lambda_t = \ln \Bigg\{ & \phi_t \mu_t \left[1 - S\left(\frac{i_t}{i_{t-1}} e^{z_t + \left(\frac{\alpha}{1-\alpha}+1\right)v_t}\right) - \frac{i_t}{i_{t-1}} e^{z_t + \left(\frac{\alpha}{1-\alpha}+1\right)v_t} S'\left(\frac{i_t}{i_{t-1}} e^{z_t + \left(\frac{\alpha}{1-\alpha}+1\right)v_t}\right)\right] + \\
& + \beta E_t \left\{ \phi_{t+1}\mu_{t+1} e^{-\left(z_{t+1} + \left(\frac{\alpha}{1-\alpha}+1\right)v_{t+1}\right)} \left(\frac{i_{t+1}}{i_t} e^{z_{t+1} + \left(\frac{\alpha}{1-\alpha}+1\right)v_{t+1}}\right)^2 S'\left(\frac{i_{t+1}}{i_t} e^{z_{t+1} + \left(\frac{\alpha}{1-\alpha}+1\right)v_{t+1}}\right)\right\} \Bigg\}
\end{aligned}$$

$$\begin{aligned}
\hat{\lambda}_t = & \frac{1}{\phi} \Bigg\{ (\phi_t - \phi) + \phi(\mu_t - 1) - \phi \frac{i}{i} e^{\gamma_z + \left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v} S'' \frac{e^{\gamma_z + \left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v}}{i} (i_t - i) + \\
& - \beta\phi e^{-\left(\gamma_z + \left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v\right)} \left(\frac{i}{i} e^{\gamma_z + \left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v}\right)^2 S'' e^{\gamma_z + \left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v} i^{-1} (i_t - i) + \\
& + \phi \frac{i}{i} e^{\gamma_z + \left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v} S'' e^{\gamma_z + \left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v} i^{-1} (i_{t-1} - i) + \\
& + \beta\phi e^{-\left(\gamma_z + \left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v\right)} \left(\frac{i}{i} e^{\gamma_z + \left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v}\right)^2 S'' e^{\gamma_z + \left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v} i^{-1} (i_{t+1} - i) + \\
& - \phi \frac{i}{i} e^{\gamma_z + \left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v} S'' e^{\gamma_z + \left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v} \frac{i}{i} (z_t - \gamma_z) + \\
& + \beta\phi e^{\gamma_z + \left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v} \left(\frac{i}{i}\right)^2 S'' e^{\gamma_z + \left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v} \frac{i}{i} (z_{t+1} - \gamma_z) + \\
& - \phi \frac{\alpha}{1-\alpha} \frac{i}{i} e^{\gamma_z + \left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v} S'' e^{\gamma_z + \left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v} \frac{i}{i} (v_t - \gamma_v) + \\
& + \frac{\alpha}{1-\alpha} \beta\phi e^{\gamma_z + \left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v} \left(\frac{i}{i}\right)^2 S'' e^{\gamma_z + \left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v} \frac{i}{i} (v_{t+1} - \gamma_v) \Bigg\}
\end{aligned}$$

which finally gives

$$\begin{aligned}
\hat{\lambda}_t = & \hat{\phi}_t + \hat{\mu}_t - (1+\beta) e^{2\left(\gamma_z + \left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v\right)} S'' \hat{\imath}_t + e^{2\left(\gamma_z + \left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v\right)} S'' \hat{\imath}_{t-1} + \\
& + \beta e^{2\left(\gamma_z + \left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v\right)} S'' E_t\{\hat{\imath}_{t+1}\} - (1 - \beta\rho_z) e^{2\left(\gamma_z + \left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v\right)} S'' \hat{z}_t + \\
& - \left(\frac{\alpha}{1-\alpha} + 1\right)(1 - \beta\rho_v) e^{2\left(\gamma_z + \left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v\right)} S'' \hat{v}_t
\end{aligned} \tag{E.9}$$

I next log-linearize the detrended optimality condition for maintenance supply given in (D.17)

$$\ln \lambda_t = \ln \Bigg\{ \left[\sigma \phi_t \left(\frac{m_t}{\bar{k}_{t-1}} e^{z_t + \left(\frac{\alpha}{1-\alpha}+1\right) v_t}\right)^{-1} \left(\delta_t - \bar{\delta}\right) e^{\sigma v_t} - \varsigma_t\right] d_t \left[1 - f\left(\frac{\tilde{m}_t}{\tilde{m}_{t-1}} e^{z_t + \frac{\alpha}{1-\alpha} v_t}\right) + \right.$$

$$\left. - \frac{\tilde{m}_t}{\tilde{m}_{t-1}} e^{z_t + \frac{\alpha}{1-\alpha} v_t} f'\left(\frac{\tilde{m}_t}{\tilde{m}_{t-1}} e^{z_t + \frac{\alpha}{1-\alpha} v_t}\right)\right] +$$

$$+ \beta E_t \Bigg\{ \left[\sigma \phi_{t+1} \left(\frac{m_{t+1}}{\bar{k}_t} e^{z_{t+1} + \left(\frac{\alpha}{1-\alpha}+1\right) v_{t+1}}\right)^{-1} \left(\delta_{t+1} - \bar{\delta}\right) e^{\sigma v_{t+1}} - \varsigma_{t+1}\right] d_{t+1} e^{-\left(z_{t+1} + \frac{\alpha}{1-\alpha} v_{t+1}\right)} \times$$

$$\times \left(\frac{\tilde{m}_{t+1}}{\tilde{m}_t} e^{z_{t+1} + \frac{\alpha}{1-\alpha} v_{t+1}}\right)^2 f'\left(\frac{\tilde{m}_{t+1}}{\tilde{m}_t} e^{z_{t+1} + \frac{\alpha}{1-\alpha} v_{t+1}}\right) \Bigg\} \Bigg\}$$

$$\hat{\lambda}_t = \frac{1}{\lambda} \Bigg\{ \sigma \phi \left(\frac{\tilde{m}}{\bar{k}} e^{\gamma_z + \left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v}\right)^{-1} \left(\delta - \bar{\delta}\right) e^{\sigma \gamma_v} \Big[\hat{\phi}_t - \hat{m}_t + \hat{\bar{k}}_{t-1} - \hat{z}_t +$$

$$- \left(\frac{\alpha}{1-\alpha} + 1 - \sigma\right) \hat{v}_t\Big] + \sigma \phi \left(\frac{\tilde{m}}{\bar{k}} e^{\gamma_z + \left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v}\right)^{-1} \delta e^{\sigma \gamma_v} \hat{\delta}_t - \varsigma \hat{\varsigma}_t +$$

$$- \left[\sigma \phi \left(\frac{\tilde{m}}{\bar{k}} e^{\gamma_z + \left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v}\right)^{-1} \left(\delta - \bar{\delta}\right) e^{\sigma \gamma_v} - \varsigma\right] \left(\frac{\tilde{m}}{\tilde{m}} e^{\gamma_z + \left(\frac{\alpha}{1-\alpha}\right)\gamma_v}\right)^2 f'' \Big[\hat{\tilde{m}}_t - \hat{\tilde{m}}_{t-1} + \hat{z}_t + \frac{\alpha}{1-\alpha} \hat{v}_t\Big] +$$

$$+ \beta \left[\sigma \phi \left(\frac{\tilde{m}}{\bar{k}} e^{\gamma_z + \left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v}\right)^{-1} \left(\delta - \bar{\delta}\right) e^{\sigma \gamma_v} - \varsigma\right] e^{-\left(\gamma_z + \frac{\alpha}{1-\alpha}\gamma_v\right)} \times$$

$$\times \left(\frac{\tilde{m}}{\tilde{m}} e^{\gamma_z + \frac{\alpha}{1-\alpha}\gamma_v}\right)^3 f'' \Big[\hat{\tilde{m}}_{t+1} - \hat{\tilde{m}}_t + \hat{z}_{t+1} + \frac{\alpha}{1-\alpha} \hat{v}_{t+1}\Big] +$$

$$+ \left[\sigma \phi \left(\frac{\tilde{m}}{\bar{k}} e^{\gamma_z + \left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v}\right)^{-1} \left(\delta - \bar{\delta}\right) e^{\sigma \gamma_v} - \varsigma\right] \hat{d}_t \Bigg\}$$

making use of the log-linearized expression of (D.16), $\hat{m}_t = \hat{d}_t + \hat{\tilde{m}}_t$, I obtain the following expression for optimal maintenance condition

$$\hat{\lambda}_t = e^{2\left(\gamma_z + \frac{\alpha}{1-\alpha}\gamma_v\right)} f'' \hat{\tilde{m}}_{t-1} + \beta e^{2\left(\gamma_z + \frac{\alpha}{1-\alpha}\gamma_v\right)} f'' E_t\{\hat{\tilde{m}}_{t+1}\} +$$

$$- \left[(1+\beta) e^{2\left(\gamma_z + \frac{\alpha}{1-\alpha}\gamma_v\right)} f'' + \sigma \left(\frac{\tilde{m}}{\bar{k}} e^{\gamma_z + \left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v}\right)^{-1} \left(\delta - \bar{\delta}\right) e^{\sigma \gamma_v}\right] \hat{\tilde{m}}_t +$$

$$+ \sigma \left(\frac{\tilde{m}}{\bar{k}} e^{\gamma_z + \left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v}\right)^{-1} \left(\delta - \bar{\delta}\right) e^{\sigma \gamma_v} \left(\hat{\phi}_t + \hat{\bar{k}}_{t-1}\right) + \sigma \left(\frac{\tilde{m}}{\bar{k}} e^{\gamma_z + \left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v}\right)^{-1} \delta e^{\sigma \gamma_v} \hat{\delta}_t +$$

$$+\left[1-\sigma\left(\frac{\tilde{m}}{\bar{k}}e^{\gamma_z+\left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v}\right)^{-1}\left(\delta-\bar{\delta}\right)e^{\sigma\gamma_v}\right]\left(\hat{d}_t+\hat{\varsigma}_t\right)+$$

$$-\left[\left(1-\beta\rho_z\right)e^{2\left(\gamma_z+\frac{\alpha}{1-\alpha}\gamma_v\right)}f''+\sigma\left(\frac{\tilde{m}}{\bar{k}}e^{\gamma_z+\left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v}\right)^{-1}\left(\delta-\bar{\delta}\right)e^{\sigma\gamma_v}\right]\hat{z}_t+$$

$$-\left[\frac{\alpha}{1-\alpha}\left(1-\beta\rho_v\right)e^{2\left(\gamma_z+\frac{\alpha}{1-\alpha}\gamma_v\right)}f''+\left(\frac{\alpha}{1-\alpha}+1-\sigma\right)\sigma\left(\frac{\tilde{m}}{\bar{k}}e^{\gamma_z+\left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v}\right)^{-1}\left(\delta-\bar{\delta}\right)e^{\sigma\gamma_v}\right]\hat{v}_t$$

and rearranging the terms I end up with

$$\begin{aligned}&\left[\left(1+\beta\right)e^{2\left(\gamma_z+\frac{\alpha}{1-\alpha}\gamma_v\right)}f''+\bar{A}\right]\hat{\tilde{m}}_t=e^{2\left(\gamma_z+\frac{\alpha}{1-\alpha}\gamma_v\right)}f''\hat{\tilde{m}}_{t-1}+\beta e^{2\left(\gamma_z+\frac{\alpha}{1-\alpha}\gamma_v\right)}f''E_t\{\hat{\tilde{m}}_{t+1}\}+\bar{A}\hat{\tilde{k}}_{t-1}+\\&+\bar{A}\frac{\delta}{\delta-\bar{\delta}}\hat{\delta}_t-\left(\bar{A}-1\right)\hat{d}_t-\left(\bar{A}-1\right)\left(\hat{\varsigma}_t-\hat{\lambda}_t\right)+\bar{A}\left(\hat{\phi}_t-\hat{\lambda}_t\right)+\\&-\left[\left(1-\beta\rho_z\right)e^{2\left(\gamma_z+\frac{\alpha}{1-\alpha}\gamma_v\right)}f''+\bar{A}\right]\hat{z}_t-\left[\frac{\alpha}{1-\alpha}\left(1-\beta\rho_v\right)e^{2\left(\gamma_z+\frac{\alpha}{1-\alpha}\gamma_v\right)}f''+\left(\frac{\alpha}{1-\alpha}+1-\sigma\right)\bar{A}\right]\hat{v}_t\end{aligned} \tag{E.10}$$

where the constant $\bar{A}=\sigma\left(\frac{\tilde{m}}{\bar{k}}e^{\gamma_z+\left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v}\right)^{-1}\left(\delta-\bar{\delta}\right)e^{\sigma\gamma_v}\geq 0$ equals the steady state expression of the depreciation rate first order condition with respect to maintenance to effective capital ratio, that is $-\delta_{m/k}e^{\sigma\gamma_v}$, and $\hat{q}_t=\hat{\phi}_t-\hat{\lambda}_t$ is the Tobin's q. According to the above expression, current expenses in maintenance increase when the rate of depreciation accelerates or, else, when the price of new investment becomes relatively high. Conversely, a rise in the current price of maintenance reduces $\hat{\tilde{m}}_t$. In response to a positive MST shock $\hat{\tilde{m}}_t$ declines as well as far as an acceleration in MST progress increases the amount of maintenance expressed in efficiency units, whereas decreases the amount of real maintenance in consumption units.

Next, it follows the log-linearization of the detrended optimal condition for the capital utilization rate, (D.19), making use of (D.20). Re-express, first, both the equations as follows, respectively, making use of (D.18) for the second one

$$\begin{aligned}&a'\left(u_t\right)=\rho_t-\eta\frac{\phi_t}{\lambda_t}\left(\delta_t-\bar{\delta}\right)e^{\sigma v_t}u_t^{-1}+\tau\frac{\varsigma_t}{\lambda_t}\\&a'(1)=\rho-\eta\frac{\phi}{\lambda}\left(\delta-\bar{\delta}\right)e^{\sigma\gamma_v}+\tau\left[\sigma\frac{\phi}{\lambda}\left(\frac{\tilde{m}}{\bar{k}}e^{\gamma_z+\left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v}\right)^{-1}\left(\delta-\bar{\delta}\right)e^{\sigma\gamma_v}-1\right]\end{aligned} \tag{E.11}$$

Call $\chi=a''(1)/a'(1)$ the value of the relative elasticity of the utilization costs of capital in steady state, and log-linearize (D.19) as follows

$$\ln\left[a'\left(u_t\right)\right]=\ln\left[\rho_t-\eta\frac{\phi_t}{\lambda_t}\left(\delta_t-\bar{\delta}\right)e^{\sigma v_t}u_t^{-1}+\tau\frac{\varsigma_t}{\lambda_t}\right]$$

$$\frac{a''(1)}{a'(1)}\hat{u}_t = \frac{1}{a'(u)}\left\{\rho\hat{\rho}_t - \eta\frac{\phi}{\lambda}\left(\delta - \bar{\delta}\right)e^{\sigma\gamma_v}\left[\hat{\phi}_t - \hat{\lambda}_t - \hat{u}_t + \sigma\hat{v}_t\right] - \eta\frac{\phi}{\lambda}\delta e^{\sigma\gamma_v}\hat{\delta}_t + \tau\frac{\varsigma}{\lambda}\left(\hat{\varsigma}_t - \hat{\lambda}_t\right)\right\}$$

$$\left\{\chi\left\{\rho - \eta\frac{\phi}{\lambda}\left(\delta - \bar{\delta}\right)e^{\sigma\gamma_v} + \tau\left[\sigma\frac{\phi}{\lambda}\left(\frac{\tilde{m}}{\bar{k}}e^{\gamma_z + \left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v}\right)^{-1}\left(\delta - \bar{\delta}\right)e^{\sigma\gamma_v} - 1\right]\right\} - \eta\frac{\phi}{\lambda}\left(\delta - \bar{\delta}\right)e^{\sigma\gamma_v}\right\}\hat{u}_t =$$

$$= \rho\hat{\rho}_t - \eta\frac{\phi}{\lambda}\left(\delta - \bar{\delta}\right)e^{\sigma\gamma_v}\left[\hat{\phi}_t - \hat{\lambda}_t + \sigma\hat{v}_t\right] - \eta\frac{\phi}{\lambda}\delta e^{\sigma\gamma_v}\hat{\delta}_t +$$

$$+ \tau\left[\sigma\frac{\phi}{\lambda}\left(\frac{\tilde{m}}{\bar{k}}e^{\gamma_z + \left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v}\right)^{-1}\left(\delta - \bar{\delta}\right)e^{\sigma\gamma_v} - 1\right]\left(\hat{\varsigma}_t - \hat{\lambda}_t\right)$$

Recalling that, in steady state, $\lambda = \phi$, the latter expression becomes

$$\hat{\rho}_t = \left\{\frac{\chi}{\rho}\left[\rho - \bar{B} + \tau\left(\bar{A} - 1\right)\right] - \frac{\bar{B}}{\rho}\right\}\hat{u}_t + \frac{\sigma}{\rho}\bar{B}\hat{v}_t + \frac{\bar{B}}{\rho}\left(\hat{\phi}_t - \hat{\lambda}_t\right) - \frac{\tau}{\rho}\left(\bar{A} - 1\right)\left(\hat{\varsigma}_t - \hat{\lambda}_t\right) + \frac{\delta}{\rho\left(\delta - \bar{\delta}\right)}\bar{B}\hat{\delta}_t \tag{E.12}$$

where the constant $\bar{B} = \eta e^{\sigma\gamma_v}\left(\delta - \bar{\delta}\right) \geq 0$ is the steady state first order condition of capital depreciation with respect to utilization rate, that is $\delta_u e^{\sigma\gamma_v}$. The latter expression defines the log-linearized optimal demand for capital utilization rate, which determines the convergence path for the rental price of capital. Note that, differently from Justiniano et al. (2011) where the convergence path of $\hat{\rho}_t$ is described by the capital utilization rate only, in this model capital rental price is positively related, among others, to the capital depreciation rate, $\hat{\delta}_t$. The current relative shadow value of maintenance, $\hat{\varsigma}_t - \hat{\lambda}_t$, impacts negatively on $\hat{\rho}_t$ as far as it can be shown that in equilibrium $\bar{A}$ is greater than one. Therefore, an increase in $\hat{\varsigma}_t$ tears down the level of maintenance rendering capital stock less attractive and thus less worthy. Tobin's q appears to impact positively on the marginal return of capital. The effect of the capital utilization rate, instead, is ambiguous and depends crucially, among others, on the deep parameters of the capital depreciation rate function.

The detrended optimal demand for capital stock in (D.21), using its steady state expression (D.22), is log-linearized below

$$\ln\phi_t = \ln\left\{\beta E_t\left\{\lambda_{t+1}\left[\rho_{t+1}u_{t+1} - a\left(u_{t+1}\right)\right]e^{-\left(z_{t+1} + \left(\frac{\alpha}{1-\alpha}+1\right)v_{t+1}\right)} +\right.\right.$$

$$+ \phi_{t+1}\left(1 - \delta_{t+1}e^{\sigma v_{t+1}}\right)e^{-\left(z_{t+1} + \left(\frac{\alpha}{1-\alpha}+1\right)v_{t+1}\right)} + \tau\varsigma_{t+1}u_{t+1}e^{-\left(z_{t+1} + \left(\frac{\alpha}{1-\alpha}+1\right)v_{t+1}\right)} +$$

$$\left.\left. - \sigma\phi_{t+1}\left(\delta_{t+1} - \bar{\delta}\right)e^{\sigma v_{t+1}}e^{-\left(z_{t+1} + \left(\frac{\alpha}{1-\alpha}+1\right)v_{t+1}\right)}\right\}\right\}$$

$$\hat{\phi}_t = \frac{1}{\phi}\left\{\beta e^{-\left(\gamma_z + \left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v\right)}\left[\lambda\rho + \phi\left(1 - \delta e^{\sigma\gamma_v}\right) - \phi\sigma\left(\delta - \bar{\delta}\right)e^{\sigma\gamma_v} + \tau\varsigma\right]\left[-\hat{z}_{t+1} - \left(\frac{\alpha}{1-\alpha} + 1\right)\hat{v}_{t+1}\right] +\right.$$

$$+\beta\lambda\rho e^{-\left(\gamma_z+\left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v\right)}\left[\hat{\lambda}_{t+1}+\hat{\rho}_{t+1}+\hat{u}_{t+1}\right]-\beta\lambda a'(1)e^{-\left(\gamma_z+\left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v\right)}\hat{u}_{t+1}+$$

$$+\beta\phi\left(1-\delta e^{\sigma\gamma_v}\right)e^{-\left(\gamma_z+\left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v\right)}\hat{\phi}_{t+1}-\beta\phi\delta e^{\sigma\gamma_v}e^{-\left(\gamma_z+\left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v\right)}\left[\hat{\delta}_{t+1}+\sigma\hat{v}_{t+1}\right]+$$

$$-\beta\sigma\phi\delta e^{\sigma\gamma_v}e^{-\left(\gamma_z+\left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v\right)}\hat{\delta}_{t+1}-\beta\sigma\phi\left(\delta-\bar{\delta}\right)e^{\sigma\gamma_v}e^{-\left(\gamma_z+\left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v\right)}\left(\hat{\phi}_{t+1}+\sigma\hat{v}_{t+1}\right)+$$

$$\left.+\beta\tau\varsigma e^{-\left(\gamma_z+\left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v\right)}\left[\hat{\varsigma}_{t+1}+\hat{u}_{t+1}\right]\right\}$$

$$\hat{\phi}_t=-\hat{z}_{t+1}-\left[\left(\frac{\alpha}{1-\alpha}+1\right)+\beta\delta\sigma e^{\sigma\gamma_v}e^{-\left(\gamma_z+\left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v\right)}+\beta\sigma^2\left(\delta-\bar{\delta}\right)e^{\sigma\gamma_v}e^{-\left(\gamma_z+\left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v\right)}\right]\hat{v}_{t+1}+$$

$$+\frac{\rho}{\rho-\tau}\left\{1-\beta e^{-\gamma_z-\left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v}\left(1-\delta e^{\sigma\gamma_v}\right)+\right.$$

$$\left.+\beta e^{-\left(\gamma_z+\left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v\right)}\sigma\left(\delta-\bar{\delta}\right)e^{\sigma\gamma_v}\left[1-\tau\left(\frac{\tilde{m}}{\bar{k}}e^{\gamma_z+\left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v}\right)^{-1}\right]\right\}\left[\hat{\lambda}_{t+1}+\hat{\rho}_{t+1}\right]+$$

$$+\beta e^{-\left(\gamma_z+\left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v\right)}\left[\left(1-\delta e^{\sigma\gamma_v}\right)-\sigma\left(\delta-\bar{\delta}\right)e^{\sigma\gamma_v}\right]\hat{\phi}_{t+1}+$$

$$-\beta e^{-\left(\gamma_z+\left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v\right)}\left[\delta e^{\sigma\gamma_v}+\sigma\delta e^{\sigma\gamma_v}\right]\hat{\delta}_{t+1}+$$

$$+\beta e^{-\left(\gamma_z+\left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v\right)}\tau\left[\sigma\left(\frac{\tilde{m}}{\bar{k}}e^{\gamma_z+\left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v}\right)^{-1}\left(\delta-\bar{\delta}\right)e^{\sigma\gamma_v}-\frac{\lambda}{\phi}\right]\hat{\varsigma}_{t+1}+$$

$$+\beta e^{-\left(\gamma_z+\left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v\right)}\left\{\eta\left(\delta-\bar{\delta}\right)e^{\sigma\gamma_v}-\tau\left[\sigma\left(\frac{\tilde{m}}{\bar{k}}e^{\gamma_z+\left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v}\right)^{-1}\left(\delta-\bar{\delta}\right)e^{\sigma\gamma_v}-\frac{\lambda}{\phi}\right]\right\}\hat{u}_{t+1}$$

$$+\beta e^{-\left(\gamma_z+\left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v\right)}\tau\left[\sigma\left(\frac{\tilde{m}}{\bar{k}}e^{\gamma_z+\left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v}\right)^{-1}\left(\delta-\bar{\delta}\right)e^{\sigma\gamma_v}-\frac{\lambda}{\phi}\right]\hat{u}_{t+1}$$

Rearranging the latter expression I obtain the following log-linear optimal capital stock condition

$$\begin{aligned}\hat{\phi}_t=&E_t\{\hat{\phi}_{t+1}\}-\rho_z\hat{z}_t-\left[\frac{\alpha}{1-\alpha}+1+\beta e^{-\left(\gamma_z+\left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v\right)}\sigma\bar{C}\right]\rho_v\hat{v}_t+\beta e^{-\left(\gamma_z+\left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v\right)}\bar{B}E_t\{\hat{u}_{t+1}\}+\\&-\beta e^{-\left(\gamma_z+\left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v\right)}\left(\bar{C}+e^{\sigma\gamma_v}\sigma\bar{\delta}\right)\hat{\delta}_{t+1}-\left[\bar{D}+\beta e^{-\left(\gamma_z+\left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v\right)}\tau\bar{A}\right]E_t\{\hat{\phi}_{t+1}-\hat{\lambda}_{t+1}\}+\\&-\left[\beta e^{-\left(\gamma_z+\left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v\right)}\tau\bar{A}-\frac{\tau}{\rho-\tau}\bar{D}\right]E_t\{\hat{\varsigma}_{t+1}-\hat{\lambda}_{t+1}\}+\frac{\rho}{\rho-\tau}\bar{D}E_t\{\hat{\rho}_{t+1}\}\end{aligned}\tag{E.13}$$

where the constants have been defined as follows

$$\bar{C}=e^{\sigma\gamma_v}\left[\delta+\sigma\left(\delta-\bar{\delta}\right)\right]>0$$

$$\bar{D} = 1 - \beta e^{-\left(\gamma_z + \left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v\right)}\left(1 - e^{\sigma\gamma_v}\delta\right) + \beta e^{-\left(\gamma_z + \left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v\right)}\frac{\sigma}{\eta}\bar{B} - \beta e^{-\left(\gamma_z + \left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v\right)}\tau\bar{A} > 0$$

The constant $\bar{D}$ represents the steady state marginal product of capital net of the marginal propensity to maintain, i.e. $\beta e^{-\left(\gamma_z + \left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v\right)}(\rho - \tau)$.
Equation (E.13) describes the optimal convergence path of the demand for new physical capital. Accordingly, when depreciation rate is expected to rise, the current shadow price of installed capital, $\hat{\phi}_t$, decreases. The same occurs when expected relative shadow value of maintenance, $\left(\hat{\varsigma}_{t+1} - \hat{\lambda}_{t+1}\right)$, increases as far as it can be shown that, in equilibrium, the term in squared brackets is positive. The expected rate of utilization, $\hat{u}_{t+1}$, impacts positively on the current shadow value of capital as far as more capital stock is expected to be used in the production process. Similarly, an increase in the expected rental price of capital, $\hat{\rho}_{t+1}$, rises $\hat{\phi}_t$. A rise in the expected Tobin's $\hat{q}_{t+1} = \hat{\phi}_{t+1} - \hat{\lambda}_{t+1}$, boosts current investment in new capital to the detriment of future investments, which lowers the respective current price.

The detrended optimal wage setting condition (D.23) is log-linearized as follows

$$\ln\left\{E_t \sum_{s=0}^{\infty} \xi_w^s \beta^s \lambda_{t+s} \tilde{L}_{t+s}(j) \tilde{w}_t(j) \tilde{\pi}_{t,t+s}^w\right\} = \ln\left\{E_t \sum_{s=0}^{\infty} \xi_w^s \beta^s \lambda_{t+s} \tilde{L}_{t+s}(j) b_{t+s} \varphi\left(1 + \lambda_{w,t+s}\right) \frac{\tilde{L}_{t+s}(j)^{\nu}}{\lambda_{t+s}}\right\}$$

$$\ln\left\{E_t \sum_{s=0}^{\infty} \xi_w^s \beta^s \tilde{w}_t(j) \tilde{\pi}_{t,t+s}^w\right\} = \ln\left\{E_t \sum_{s=0}^{\infty} \xi_w^s \beta^s b_{t+s} \varphi\left(1 + \lambda_{w,t+s}\right) \frac{\tilde{L}_{t+s}(j)^{\nu}}{\lambda_{t+s}}\right\}$$

$$E_t \sum_{s=0}^{\infty} \xi_w^s \beta^s \left[\ln \tilde{w}_t(j) + \ln \tilde{\pi}_{t,t+s}^w\right] = E_t \sum_{s=0}^{\infty} \xi_w^s \beta^s \left[\ln b_{t+s} + \ln\varphi + \ln\left(1 + \lambda_{w,t+s}\right) + \ln \tilde{L}_{t+s}(j)^{\nu} - \ln \lambda_{t+s}\right]$$

obtaining

$$\frac{1}{1 - \xi_w \beta} \hat{\tilde{w}}_t(j) = E_t \sum_{s=0}^{\infty} \xi_w^s \beta^s \left[\hat{b}_{t+s} + \hat{\lambda}_{w,t+s} + \nu \hat{\tilde{L}}_{t+s}(j) - \hat{\lambda}_{t+s} - \hat{\tilde{\pi}}_{t,t+s}^w\right] \tag{E.14}$$

Next, I log-linearize as follows the definition of wage inflation

$$\ln \tilde{\pi}_{t,t+s}^w = \ln\left\{\prod_{k=0}^{s}\left[\left(\frac{\pi_{t+k-1} e^{z_{t+k-1} + \frac{\alpha}{1-\alpha} v_{t+k-1}}}{\pi e^{\gamma_z + \frac{\alpha}{1-\alpha}\gamma_v}}\right)^{\iota_w}\left(\frac{\pi_{t+k} e^{z_{t+k} + \frac{\alpha}{1-\alpha} v_{t+k}}}{\pi e^{\gamma_z + \frac{\alpha}{1-\alpha}\gamma_v}}\right)^{-1}\right]\right\}$$

$$\ln \tilde{\pi}_{t,t+s}^w = \sum_{k=0}^{s}\left\{\iota_w \ln \pi_{t+k-1} + \iota_w\left(z_{t+k-1} + \frac{\alpha}{1-\alpha} v_{t+k-1}\right) - \iota_w \ln\pi - \iota_w\left(\gamma_z + \frac{\alpha}{1-\alpha}\gamma_v\right) + \right.$$

$$\left. - \left(z_{t+k} + \frac{\alpha}{1-\alpha} v_{t+k}\right) + \ln\pi + \left(\gamma_z + \frac{\alpha}{1-\alpha}\gamma_v\right) - \ln \pi_{t+k}\right\}$$

which gives

$$\hat{\tilde{\pi}}^w_{t,t+s} = \sum_{k=0}^{s}\left[\iota_w\left(\hat{\pi}_{t+k-1} + \hat{z}_{t+k-1} + \frac{\alpha}{1-\alpha}\hat{v}_{t+k-1}\right) - \left(\hat{\pi}_{t+k} + \hat{z}_{t+k} + \frac{\alpha}{1-\alpha}\hat{v}_{t+k}\right)\right] \tag{E.15}$$

The log-linearized detrended labor demand, (D.25), becomes

$$\ln L_{t+s}(j) = \ln\left\{\left[\frac{w_t(j)}{w_{t+s}}\tilde{\pi}^w_{t,t+s}\right]^{-\frac{1+\lambda_{w,t+s}}{\lambda_{w,t+s}}} L_{t+s}\right\}$$

$$\ln L_{t+s}(j) = -\frac{1+\lambda_{w,t+s}}{\lambda_{w,t+s}}\ln\left[\frac{w_t(j)}{w_{t+s}}\tilde{\pi}^w_{t,t+s}\right] + \ln L_{t+s}$$

therefore,

$$\hat{L}_{t+s}(j) = -\frac{1+\lambda_w}{\lambda_w}\left[\hat{\tilde{w}}_t(j) - \hat{w}_{t+s} + \hat{\tilde{\pi}}^w_{t,t+s}\right] + \hat{L}_{t+s} \tag{E.16}$$

Now, combine equations (E.14) and (E.16) as follows

$$\frac{1}{1-\xi_w\beta}\hat{\tilde{w}}_t(j) = E_t\sum_{s=0}^{\infty}\xi_w^s\beta^s\left\{\hat{b}_{t+s} + \hat{\lambda}_{w,t+s} - \hat{\lambda}_{t+s} - \hat{\tilde{\pi}}^w_{t,t+s} + \nu\left[\hat{L}_{t+s} - \frac{1+\lambda_w}{\lambda_w}\left(\hat{\tilde{w}}_t(j) - \hat{w}_{t+s} + \hat{\tilde{\pi}}^w_{t,t+s}\right)\right]\right\}$$

$$\frac{1}{1-\xi_w\beta}\left(1+\nu\frac{1+\lambda_w}{\lambda_w}\right)\hat{\tilde{w}}_t(j) = E_t\sum_{s=0}^{\infty}\xi_w^s\beta^s\Big[\nu\hat{L}_{t+s} + \nu\frac{1+\lambda_w}{\lambda_w}\hat{w}_{t+s} + \hat{b}_{t+s} + \hat{\lambda}_{w,t+s} - \hat{\lambda}_{t+s} +$$

$$-\left(1+\nu\frac{1+\lambda_w}{\lambda_w}\right)\hat{\tilde{\pi}}^w_{t,t+s}\Big] =$$

$$= \xi_w^0\beta^0\left[\nu\hat{L}_t + \nu\frac{1+\lambda_w}{\lambda_w}\hat{w}_t + \hat{b}_t + \hat{\lambda}_{w,t} - \hat{\lambda}_t - \left(1+\nu\frac{1+\lambda_w}{\lambda_w}\right)\hat{\tilde{\pi}}^w_{t,t}\right] +$$

$$+ E_t\sum_{s=1}^{\infty}\xi_w^s\beta^s\left[\nu\hat{L}_{t+s} + \nu\frac{1+\lambda_w}{\lambda_w}\hat{w}_{t+s} + \hat{b}_{t+s} + \hat{\lambda}_{w,t+s} - \hat{\lambda}_{t+s} - \left(1+\nu\frac{1+\lambda_w}{\lambda_w}\right)\hat{\tilde{\pi}}^w_{t,t+s}\right] =$$

$$= \nu\hat{L}_t + \nu\frac{1+\lambda_w}{\lambda_w}\hat{w}_t + \hat{b}_t + \hat{\lambda}_{w,t} - \hat{\lambda}_t + \xi_w\beta E_t\sum_{s=0}^{\infty}\xi_w^s\beta^s\Big[\nu\hat{L}_{t+s+1} + \nu\frac{1+\lambda_w}{\lambda_w}\hat{w}_{t+s+1} + \hat{b}_{t+s+1} + \hat{\lambda}_{w,t+s+1} +$$

$$-\hat{\lambda}_{t+s+1} - \left(1+\nu\frac{1+\lambda_w}{\lambda_w}\right)\hat{\tilde{\pi}}^w_{t+1,t+s+1}\Big] - \xi_w\beta E_t\sum_{s=0}^{\infty}\xi_w^s\beta^s\left(1+\nu\frac{1+\lambda_w}{\lambda_w}\right)\hat{\tilde{\pi}}^w_{t,t+1} =$$

$$= \nu\hat{L}_t + \nu\frac{1+\lambda_w}{\lambda_w}\hat{w}_t + \hat{b}_t + \hat{\lambda}_{w,t} - \hat{\lambda}_t + \frac{\xi_w\beta}{1-\xi_w\beta}\left(1+\nu\frac{1+\lambda_w}{\lambda_w}\right)\hat{\tilde{w}}_{t+1}(j) - \frac{\xi_w\beta}{1-\xi_w\beta}\left(1+\nu\frac{1+\lambda_w}{\lambda_w}\right)\hat{\tilde{\pi}}^w_{t,t+1}$$

gives

$$\frac{1}{1-\xi_w\beta}\left(1+\nu\frac{1+\lambda_w}{\lambda_w}\right)\hat{\tilde{w}}_t(j)=\frac{\xi_w\beta}{1-\xi_w\beta}\left(1+\nu\frac{1+\lambda_w}{\lambda_w}\right)E_t\{\hat{\tilde{w}}_{t+1}(j)-\hat{\tilde{\pi}}^w_{t,t+1}\}+$$
$$+\nu\hat{L}_t+\nu\frac{1+\lambda_w}{\lambda_w}\hat{w}_t+\hat{b}_t+\hat{\lambda}_{w,t}-\hat{\lambda}_t \tag{E.17}$$

The latter expression characterizes the optimal wage setting in terms of the optimal demand for labor.

The log-linearized aggregate wage index in (D.26) is given by

$$-\frac{1}{\lambda_w}\ln w_t=\ln\left\{\xi_w\left[w_{t-1}\left(\frac{\pi_{t-1}e^{z_{t-1}+\frac{\alpha}{1-\alpha}v_{t-1}}}{\pi e^{\gamma_z+\frac{\alpha}{1-\alpha}\gamma_v}}\right)^{\iota_w}\left(\frac{\pi_t e^{z_t+\frac{\alpha}{1-\alpha}v_t}}{\pi e^{\gamma_z+\frac{\alpha}{1-\alpha}\gamma_v}}\right)^{-1}\right]^{-\frac{1}{\lambda_{w,t}}}+(1-\xi_w)\,\tilde{w}_t(j)^{-\frac{1}{\lambda_{w,t}}}\right\}$$

$$-\frac{1}{\lambda_w}\hat{w}_t=\frac{1}{w^{-1/\lambda_w}}\Bigg\{-\xi_w\frac{1}{\lambda_w}w^{-\frac{1}{\lambda_w}-1}\Big[(w_{t-1}-w)+w\iota_w\frac{\pi_{t-1}-\pi}{\pi}-w\frac{\pi_t-\pi}{\pi}+w\iota_w\,(z_{t-1}-\gamma_z)+$$
$$-w\,(z_t-\gamma_z)+w\iota_w\frac{\alpha}{1-\alpha}\,(v_{t-1}-\gamma_v)-w\frac{\alpha}{1-\alpha}\,(v_t-\gamma_v)\Big]-\frac{1}{\lambda_w}\,(1-\xi_w)\,w^{-\frac{1}{\lambda_w}-1}\,[\tilde{w}_t(j)-w]\Bigg\}$$

$$\hat{w}_t=(1-\xi_w)\,\hat{\tilde{w}}_t(j)+\xi_w\left[\hat{w}_{t-1}-\hat{\pi}_t-\hat{z}_t-\frac{\alpha}{1-\alpha}\hat{v}_t+\iota_w\hat{\pi}_{t-1}+\iota_w\hat{z}_{t-1}+\iota_w\frac{\alpha}{1-\alpha}\hat{v}_{t-1}\right] \tag{E.18}$$

Combine the above expression with the log-linearized wage inflation given in (E.15) in order to obtain the following log-linearized aggregate wage index in terms of wage inflation

$$\hat{w}_t=(1-\xi_w)\,\hat{\tilde{w}}_t(j)+\xi_w\left[\hat{w}_{t-1}+\hat{\tilde{\pi}}^w_{t-1,t}\right] \tag{E.19}$$

Next, combine equation (E.17) with equation (E.19) expressed both in current period and forward period, as follows

$$\frac{1}{1-\xi_w\beta}\left(1+\nu\frac{1+\lambda_w}{\lambda_w}\right)\left[\frac{1}{1-\xi_w}\hat{w}_t-\frac{\xi_w}{1-\xi_w}\hat{w}_{t-1}-\frac{\xi_w}{1-\xi_w}\hat{\tilde{\pi}}^w_{t-1,t}\right]=$$
$$=\frac{\xi_w\beta}{1-\xi_w\beta}\left(1+\nu\frac{1+\lambda_w}{\lambda_w}\right)\left[\frac{1}{1-\xi_w}E_t\{\hat{w}_{t+1}\}-\frac{\xi_w}{1-\xi_w}\hat{w}_t-\frac{\xi_w}{1-\xi_w}E_t\{\hat{\tilde{\pi}}^w_{t,t+1}\}-E_t\{\hat{\tilde{\pi}}^w_{t,t+1}\}\right]+$$
$$+\nu\hat{L}_t+\nu\frac{1+\lambda_w}{\lambda_w}\hat{w}_t+\hat{b}_t+\hat{\lambda}_{w,t}-\hat{\lambda}_t$$

Collect the terms, and add $-\hat{w}_t$ on both the sides of the above equation

$$\left(1+\nu\frac{1+\lambda_w}{\lambda_w}\right)\left[\frac{1}{(1-\xi_w\beta)(1-\xi_w)}+\frac{\xi_w^2\beta}{(1-\xi_w\beta)(1-\xi_w)}\right]\hat{w}_t-\nu\frac{1+\lambda_w}{\lambda_w}\hat{w}_t-\hat{w}_t=$$

$$=-\hat{w}_t+\left(1+\nu\frac{1+\lambda_w}{\lambda_w}\right)\frac{\xi_w}{(1-\xi_w\beta)(1-\xi_w)}\hat{w}_{t-1}+\left(1+\nu\frac{1+\lambda_w}{\lambda_w}\right)\frac{\xi_w\beta}{(1-\xi_w\beta)(1-\xi_w)}\hat{w}_{t+1}+$$

$$+\left(1+\nu\frac{1+\lambda_w}{\lambda_w}\right)\frac{\xi_w}{(1-\xi_w\beta)(1-\xi_w)}\hat{\tilde{\pi}}^w_{t-1,t}-\left(1+\nu\frac{1+\lambda_w}{\lambda_w}\right)\frac{\xi_w\beta}{1-\xi_w\beta}\left(\frac{\xi_w}{1-\xi_w}+1\right)\hat{\tilde{\pi}}^w_{t,t+1}+$$

$$+\left(\nu\hat{L}_t+\hat{b}_t-\hat{\lambda}_t\right)+\hat{\lambda}_{w,t}$$

$$\left(1+\nu\frac{1+\lambda_w}{\lambda_w}\right)\frac{\xi_w(1+\beta)}{(1-\xi_w\beta)(1-\xi_w)}\hat{w}_t=-\left[\hat{w}_t-\left(\nu\hat{L}_t+\hat{b}_t-\hat{\lambda}_t\right)\right]+\hat{\lambda}_{w,t}+$$

$$+\left(1+\nu\frac{1+\lambda_w}{\lambda_w}\right)\frac{\xi_w}{(1-\xi_w\beta)(1-\xi_w)}\left[\hat{w}_{t-1}+\beta\hat{w}_{t+1}+\hat{\tilde{\pi}}^w_{t-1,t}-\beta\hat{\tilde{\pi}}^w_{t,t+1}\right]$$

$$\hat{w}_t=\frac{1}{1+\beta}\left(\hat{w}_{t-1}+\hat{\tilde{\pi}}^w_{t-1,t}\right)+\frac{\beta}{1+\beta}\left(\hat{w}_{t+1}-\hat{\tilde{\pi}}^w_{t,t+1}\right)+$$

$$-\frac{1}{1+\nu\frac{1+\lambda_w}{\lambda_w}}\frac{(1-\xi_w\beta)(1-\xi_w)}{\xi_w(1+\beta)}\left[\hat{w}_t-\left(\nu\hat{L}_t+\hat{b}_t-\hat{\lambda}_t\right)-\hat{\lambda}_{w,t}\right]$$

Substitute in the above expression equation (E.15) for log-linearized wage inflation, so that

$$\hat{w}_t=\frac{1}{1+\beta}\hat{w}_{t-1}+\frac{\iota_w}{1+\beta}\left(\hat{\pi}_{t-1}+\hat{z}_{t-1}+\frac{\alpha}{1-\alpha}\hat{v}_{t-1}\right)-\frac{1}{1+\beta}\left(\hat{\pi}_t+\hat{z}_t+\frac{\alpha}{1-\alpha}\hat{v}_t\right)+$$

$$+\frac{\beta}{1+\beta}E_t\{\hat{w}_{t+1}\}-\frac{\beta\iota_w}{1+\beta}\left(\hat{\pi}_t+\hat{z}_t+\frac{\alpha}{1-\alpha}\hat{v}_t\right)+\frac{\beta}{1+\beta}E_t\{\hat{\pi}_{t+1}+\hat{z}_{t+1}+\frac{\alpha}{1-\alpha}\hat{v}_{t+1}\}+$$

$$-\frac{1}{1+\nu\frac{1+\lambda_w}{\lambda_w}}\frac{(1-\xi_w\beta)(1-\xi_w)}{\xi_w(1+\beta)}\left[\hat{w}_t-\left(\nu\hat{L}_t+\hat{b}_t-\hat{\lambda}_t\right)-\hat{\lambda}_{w,t}\right]$$

and rearrange the terms in order to obtain

$$\hat{w}_t=\frac{1}{1+\beta}\hat{w}_{t-1}+\frac{\beta}{1+\beta}E_t\{\hat{w}_{t+1}\}-\frac{1}{1+\nu\frac{1+\lambda_w}{\lambda_w}}\frac{(1-\xi_w\beta)(1-\xi_w)}{\xi_w(1+\beta)}\left[\hat{w}_t-\left(\nu\hat{L}_t+\hat{b}_t-\hat{\lambda}_t\right)\right]+$$

$$+\frac{\iota_w}{1+\beta}\hat{\pi}_{t-1}+\frac{\beta}{1+\beta}E_t\{\hat{\pi}_{t+1}\}-\frac{1+\beta\iota_w}{1+\beta}\hat{\pi}_t+\frac{\iota_w}{1+\beta}\hat{z}_{t-1}-\frac{1+\beta\iota_w-\beta\rho_z}{1+\beta}\hat{z}_t+ \tag{E.20}$$

$$+\frac{\iota_w}{1+\beta}\frac{\alpha}{1-\alpha}\hat{v}_{t-1}-\frac{1+\beta\iota_w-\beta\rho_v}{1+\beta}\frac{\alpha}{1-\alpha}\hat{v}_t+\frac{1}{1+\nu\frac{1+\lambda_w}{\lambda_w}}\frac{(1-\xi_w\beta)(1-\xi_w)}{\xi_w(1+\beta)}\hat{\lambda}_{w,t}$$

The above expression represents the log-linearized wage Phillips curve, and the term $\hat{w}_t - \left(\nu\hat{L}_t+\hat{b}_t-\hat{\lambda}_t\right)=\hat{g}_{w,t}$ is the log-linearized marginal utility of labor.

Now, re-express the wage Phillips curve in terms of the optimally chosen wage by the households, $\tilde{w}_t$. For this purpose, combine equations (E.17) and (E.15) and rearrange the terms

$$\frac{1}{1-\xi_w\beta}\left(1+\nu\frac{1+\lambda_w}{\lambda_w}\right)\hat{\tilde{w}}_t(j)=\nu\frac{1+\lambda_w}{\lambda_w}\hat{w}_t+\nu\hat{L}_t+\hat{b}_t-\hat{\lambda}_t+\hat{\lambda}_{w,t}+$$
$$+\frac{\xi_w\beta}{1-\xi_w\beta}\left(1+\nu\frac{1+\lambda_w}{\lambda_w}\right)\left(E_t\{\hat{\tilde{w}}_{t+1}(j)\}-\iota_w\hat{\pi}_t-\iota_w\hat{z}_t-\iota_w\frac{\alpha}{1-\alpha}\hat{v}_t+E_t\{\hat{\pi}_{t+1}\}+E_t\{\hat{z}_{t+1}\}+\frac{\alpha}{1-\alpha}E_t\{\hat{v}_{t+1}\}\right)$$

obtaining thus

$$\hat{\tilde{w}}_t(j)=\xi_w\beta E_t\{\hat{\tilde{w}}_{t+1}(j)\}-\xi_w\beta\iota_w\hat{\pi}_t+\xi_w\beta E_t\{\hat{\pi}_{t+1}\}+\xi_w\beta(\rho_z-\iota_w)\hat{z}_t+\xi_w\beta(\rho_v-\iota_w)\frac{\alpha}{1-\alpha}\hat{v}_t+$$
$$+\frac{1-\xi_w\beta}{1+\nu\frac{1+\lambda_w}{\lambda_w}}\left(\nu\frac{1+\lambda_w}{\lambda_w}\hat{w}_t+\nu\hat{L}_t+\hat{b}_t-\hat{\lambda}_t+\hat{\lambda}_{w,t}\right)\tag{E.21}$$

Given the definition of detrended capital utilization, (D.27), and its steady state (D.28), log-linearize it as follows

$$\ln k_t=\ln\left[u_t\bar{k}_{t-1}e^{-\left(z_t+\left(\frac{\alpha}{1-\alpha}+1\right)v_t\right)}\right]$$

$$\hat{k}_t=\frac{e^{\gamma_z+\left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v}}{\bar{k}}\Big[\frac{\bar{k}}{e^{\gamma_z+\left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v}}(u_t-1)+\frac{1}{e^{\gamma_z+\left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v}}\left(\bar{k}_{t-1}-\bar{k}\right)-\frac{\bar{k}}{e^{\gamma_z+\left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v}}(z_t-\gamma_z)+$$
$$-\frac{\bar{k}}{e^{\gamma_z+\left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v}}\left(\frac{\alpha}{1-\alpha}+1\right)(v_t-\gamma_v)\Big]$$

which gives

$$\hat{k}_t=\hat{u}_t+\hat{\bar{k}}_{t-1}-\hat{z}_t-\left(\frac{\alpha}{1-\alpha}+1\right)\hat{v}_t\tag{E.22}$$

Given equations (D.29) and (D.30), and the relations $\hat{m}_t=\hat{\tilde{m}}_t+\hat{d}_t$, and $m=\tilde{m}$, capital depreciation rate is log-linearized as follows

$$\ln\delta_t=\ln\left[\zeta u_t^{\eta}\left(\frac{m_t}{\bar{k}_{t-1}}e^{z_t+\left(\frac{\alpha}{1-\alpha}+1\right)v_t}\right)^{-\sigma}+\bar{\delta}\right]$$

$$\hat{\delta}_t = \frac{\left(\frac{\tilde{m}}{\bar{k}} e^{\gamma_z + \left(\frac{\alpha}{1-\alpha}+1\right)}\right)^{\sigma}}{\zeta + \bar{\delta}\left(\frac{\tilde{m}}{\bar{k}} e^{\gamma_z + \left(\frac{\alpha}{1-\alpha}+1\right)}\right)^{\sigma}} \left\{ \zeta\eta \left(\frac{\tilde{m}}{\bar{k}} e^{\gamma_z + \left(\frac{\alpha}{1-\alpha}+1\right)}\right)^{-\sigma} \hat{u}_t + \right.$$

$$\left. - \zeta\sigma \left(\frac{\tilde{m}}{\bar{k}} e^{\gamma_z + \left(\frac{\alpha}{1-\alpha}+1\right)}\right)^{-\sigma-1} \left(\frac{\tilde{m}}{\bar{k}} e^{\gamma_z + \left(\frac{\alpha}{1-\alpha}+1\right)}\right) \left[\hat{m}_t - \hat{\bar{k}}_{t-1} + \hat{z}_t + \left(\frac{\alpha}{1-\alpha}+1\right)\hat{v}_t\right] \right.$$

which gives

$$\hat{\delta}_t = \frac{\zeta}{\zeta + \bar{\delta}\left(\frac{\tilde{m}}{\bar{k}} e^{\gamma_z + \left(\frac{\alpha}{1-\alpha}+1\right)}\right)^{\sigma}} \left[\sigma\hat{\bar{k}}_{t-1} - \sigma\hat{\tilde{m}}_t - \sigma\hat{d}_t - \sigma\hat{z}_t - \sigma\left(\frac{\alpha}{1-\alpha}+1\right)\hat{v}_t + \eta\hat{u}_t\right] \tag{E.23}$$

The definition of maintenance costs given in (D.31), using (D.32), is log-linearized as follows

$$\ln m_t = \ln\left[\tau u_t \bar{k}_{t-1} e^{-\left(z_t + \left(\frac{\alpha}{1-\alpha}+1\right)v_t\right)} + \bar{M}\right]$$

$$\hat{m}_t = \frac{1}{\tilde{m}}\tau\bar{k}e^{-\left(\gamma_z + \left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v\right)}\left[\hat{u}_t + \hat{\bar{k}}_{t-1} - \hat{z}_t - \left(\frac{\alpha}{1-\alpha}+1\right)\hat{v}_t\right]$$

Rearranging the terms and recalling that $\hat{m}_t = \hat{\tilde{m}}_t + \hat{d}_t$, it is obtained

$$\hat{\tilde{m}}_t = -\hat{d}_t + \tau\left(\frac{\tilde{m}}{\bar{k}} e^{\gamma_z + \left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v}\right)^{-1}\left[\hat{u}_t + \hat{\bar{k}}_{t-1} - \hat{z}_t - \left(\frac{\alpha}{1-\alpha}+1\right)\hat{v}_t\right] \tag{E.24}$$

According to (E.23) and (E.24), respectively, optimal capital depreciation and maintenance costs both increase when either the intensity of utilization or the amount of old capital stock increase. On the contrary, both of them respond negatively to a positive MST shock. Recall that, an acceleration in the MST shock increases the amount of maintenance expressed in efficiency units, whereas decreases the amount of real maintenance in consumption units. The optimal path of depreciation is also negatively affected by real maintenance expenses, as it is expected to be.

Next, log-linearize the law of motion of capital, given (D.33) and (D.34), as follows

$$\ln\bar{k}_t = \ln\left\{ \left(1 - \delta_t e^{\sigma v_t}\right)\bar{k}_{t-1} e^{-\left(z_t + \left(\frac{\alpha}{1-\alpha}+1\right)v_t\right)} + \mu_t\left[1 - S\left(\frac{i_t}{i_{t-1}} e^{z_t + \left(\frac{\alpha}{1-\alpha}+1\right)v_t}\right)\right] i_t \right\}$$

$$\hat{\bar{k}}_t = \frac{1}{\bar{k}}\left\{ \left(1 - \delta e^{\sigma\gamma_v}\right)\bar{k}e^{-\left(\gamma_z + \left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v\right)}\left[\hat{\bar{k}}_{t-1} - \hat{z}_t - \left(\frac{\alpha}{1-\alpha}+1\right)\hat{v}_t\right] + \right.$$

$$- \delta e^{\sigma \gamma v} \bar{k} e^{-\left(\gamma_z + \left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v\right)} \left[\hat{\delta}_t + \sigma \hat{v}_t\right] + i \left(\hat{\mu}_t + \hat{i}_t\right) \Bigg\}$$

$$\hat{\bar{k}}_t = \left(1 - \delta e^{\sigma \gamma v}\right) e^{-\left(\gamma_z + \left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v\right)} \left[\hat{\bar{k}}_{t-1} - \hat{z}_t - \left(\frac{\alpha}{1-\alpha} + 1\right) \hat{v}_t\right] - \delta e^{\sigma \gamma v} e^{-\left(\gamma_z + \left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v\right)} \left[\hat{\delta}_t + \sigma \hat{v}_t\right] +$$
$$+ \left[1 - \left(1 - \delta e^{\sigma \gamma_v}\right) e^{-\left(\gamma_z + \left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v\right)}\right] \left(\hat{\mu}_t + \hat{i}_t\right)$$

$$\begin{aligned}\hat{\bar{k}}_t &= \left(1 - \delta e^{\sigma \gamma v}\right) e^{-\left(\gamma_z + \left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v\right)} \hat{\bar{k}}_{t-1} - \delta e^{\sigma \gamma v} e^{-\left(\gamma_z + \left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v\right)} \hat{\delta}_t + \\ &+ \left[1 - \left(1 - \delta e^{\sigma \gamma_v}\right) e^{-\left(\gamma_z + \left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v\right)}\right] \left(\hat{\mu}_t + \hat{i}_t\right) - \left(1 - \delta e^{\sigma \gamma v}\right) e^{-\left(\gamma_z + \left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v\right)} \hat{z}_t + \\ &- \left[\left(1 - \delta e^{\sigma \gamma v}\right) \left(\frac{\alpha}{1-\alpha} + 1\right) + \sigma \delta e^{\sigma \gamma v}\right] e^{-\left(\gamma_z + \left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v\right)} \hat{v}_t\end{aligned} \tag{E.25}$$

Differently from the baseline model, in the maintenance model the law of motion of capital is directly influenced by the current rate of depreciation.

Log-linearize the aggregate resource constraint, given (D.35) and (D.36), as follows

$$\ln \left[c_t + i_t + a\left(u_t\right) \bar{k}_{t-1} e^{-\left(z_t + \left(\frac{\alpha}{1-\alpha}+1\right)v_t\right)} + \tilde{m}_t\right] = \ln \left[\left(1/g_t\right) y_t\right]$$

$$\hat{y}_t - \hat{g}_t = \frac{g}{y} \left[c\hat{c}_t + i\hat{i}_t + \tilde{m}\hat{\tilde{m}}_t + a'(1)\bar{k} e^{-\left(\gamma_z + \left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v\right)} \hat{u}_t\right]$$

Substituting the expression for the steady state of the optimal capital utilization rate given in (E.11), and rearranging, the latter expression becomes

$$\begin{aligned}&\frac{1}{g}\hat{y}_t - \frac{1}{g}\hat{g}_t = \frac{c}{y}\hat{c}_t + \frac{i}{y}\hat{i}_t + \frac{\tilde{m}}{y}\hat{\tilde{m}}_t + \\ &+ \left\{\rho - \eta\left(\delta - \bar{\delta}\right) e^{\sigma \gamma_v} + \tau \left[\sigma \left(\frac{\tilde{m}}{\bar{k}} e^{\gamma_z + \left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v}\right)^{-1} \left(\delta - \bar{\delta}\right) e^{\sigma \gamma_v} - 1\right]\right\} \frac{\bar{k}}{y} e^{-\left(\gamma_z + \left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v\right)} \hat{u}_t\end{aligned} \tag{E.26}$$

Before log-linearizing the definition of actual GDP, combine equations (D.35) and (D.37) in order to obtain the following expressions

$$x_t = \left(1 - 1/g_t\right) y_t + \frac{y_t}{g_t} - a\left(u_t\right) \bar{k}_{t-1} e^{-\left(z_t + \left(\frac{\alpha}{1-\alpha}+1\right)v_t\right)}$$

$$x_t = y_t - a\left(u_t\right) \bar{k}_{t-1} e^{-\left(z_t + \left(\frac{\alpha}{1-\alpha}+1\right)v_t\right)}$$

In steady state, the latter one becomes

$$x = y$$

which, recalling (E.11), will be used for the log-linearization computations as follows

$$\ln x_t = \ln \left[y_t - a\left(u_t\right) \bar{k}_{t-1} e^{-\left(z_t + \left(\frac{\alpha}{1-\alpha}+1\right) v_t\right)} \right]$$

$$\hat{x}_t = \frac{1}{y} \left[y\hat{y} - a'(1) \bar{k} e^{-\left(\gamma_z + \left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v\right)} \hat{u}_t \right]$$

$$\hat{x}_t = \hat{y} - a'(1) \frac{\bar{k}}{y} e^{-\left(\gamma_z + \left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v\right)} \hat{u}_t$$

$$\hat{x}_t = \hat{y} - \left\{ \rho - \eta\left(\delta - \bar{\delta}\right) e^{\sigma\gamma_v} + \tau \left[\sigma \left(\frac{\tilde{m}}{\bar{k}} e^{\gamma_z + \left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v} \right)^{-1} \left(\delta - \bar{\delta}\right) e^{\sigma\gamma_v} - 1 \right] \right\} \frac{\bar{k}}{y} e^{-\left(\gamma_z + \left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v\right)} \hat{u}_t \tag{E.27}$$

Finally, denoting by $x^\star$ the stationary GDP gap, the nominal interest rate rule is log-linearized as follows

$$\ln\left(\frac{R_t}{R}\right) = \ln\left\{ \left(\frac{R_{t-1}}{R}\right)^{\rho_R} \left[\left(\frac{\pi_t}{\pi}\right)^{\phi_\pi} \left(\frac{x_t}{x_t^\star}\right)^{\phi_X} \right]^{1-\rho_R} \left(\frac{x_t/x_{t-1}}{x_t^\star/x_{t-1}^\star}\right)^{\phi_{dX}} \varepsilon_{mp,t} \right\}$$

$$\ln R_t - \ln R = \rho_R\left(\ln R_{t-1} - \ln R\right) + \left(1-\rho_R\right)\left[\phi_\pi\left(\ln \pi_t - \ln \pi\right) + \phi_X\left(\ln x_t - \ln x_t^\star\right)\right] +$$

$$+ \phi_{dX}\left(\ln x_t - \ln x_{t-1} - \ln x_t^\star + \ln x_{t-1}^\star\right) + \ln \varepsilon_{mp,t}$$

$$\hat{R}_t = \rho_R \hat{R}_{t-1} + \left(1-\rho_R\right)\left(\phi_\pi \hat{\pi}_t + \phi_X \hat{x}_t - \phi_X \hat{x}_t^\star\right) + \phi_{dX}\left(\hat{x}_t - \hat{x}_{t-1} - \hat{x}_t^\star + \hat{x}_{t-1}^\star\right) + \hat{\varepsilon}_{mp,t}$$

obtaining, finally

$$\hat{R}_t = \rho_R \hat{R}_{t-1} + \left(1-\rho_R\right)\phi_\pi \hat{\pi}_t + \left[\left(1-\rho_R\right)\phi_X + \phi_{dX}\right]\hat{x}_t +$$

$$- \left[\left(1-\rho_R\right)\phi_X + \phi_{dX}\right]\hat{x}_t^\star - \phi_{dX}\hat{x}_{t-1} + \phi_{dX}\hat{x}_{t-1}^\star + \hat{\varepsilon}_{mp,t} \tag{E.28}$$

To recap, the maintenance model is composed of the following 20 linear rational expectations equations in the sticky price-wage economy

1. $\hat{y}_t = \frac{y+F}{y}\alpha\hat{k}_t + \frac{y+F}{y}(1-\alpha)\hat{L}_t$

2. $\hat{\rho}_t = \hat{w}_t - \hat{k}_t + \hat{L}_t$

3. $\hat{s}_t = (1-\alpha)\hat{w}_t + \alpha\hat{\rho}_t$

4. $\hat{\pi}_t = \frac{\iota_p}{1+\beta\iota_p}\hat{\pi}_{t-1} + \frac{\beta}{1+\beta\iota_p}E_t\{\hat{\pi}_{t+1}\} + \frac{(1-\xi_p\beta)(1-\xi_p)}{\xi_p(1+\beta\iota_p)}(\hat{s}_t + \lambda_{p,t})$

5. $$\hat{\lambda}_t = \frac{e^{\gamma_z+\frac{\alpha}{1-\alpha}\gamma_v}}{\left(e^{\gamma_z+\frac{\alpha}{1-\alpha}\gamma_v}-\beta h\right)\left(e^{\gamma_z+\frac{\alpha}{1-\alpha}\gamma_v}-h\right)}\hat{c}_{t-1} - \frac{e^{2\left(\gamma_z+\frac{\alpha}{1-\alpha}\gamma_v\right)}+\beta h^2}{\left(e^{\gamma_z+\frac{\alpha}{1-\alpha}\gamma_v}-\beta h\right)\left(e^{\gamma_z+\frac{\alpha}{1-\alpha}\gamma_v}-h\right)}\hat{c}_t +$$
$$+ \frac{e^{\gamma_z+\frac{\alpha}{1-\alpha}\gamma_v}\beta h}{\left(e^{\gamma_z+\frac{\alpha}{1-\alpha}\gamma_v}-\beta h\right)\left(e^{\gamma_z+\frac{\alpha}{1-\alpha}\gamma_v}-h\right)}E_t\{\hat{c}_{t+1}\} + \frac{e^{\gamma_z+\frac{\alpha}{1-\alpha}\gamma_v}\beta h\rho_z - e^{\gamma_z+\frac{\alpha}{1-\alpha}\gamma_v}h}{\left(e^{\gamma_z+\frac{\alpha}{1-\alpha}\gamma_v}-\beta h\right)\left(e^{\gamma_z+\frac{\alpha}{1-\alpha}\gamma_v}-h\right)}\hat{z}_t +$$
$$+ \frac{e^{\gamma_z+\frac{\alpha}{1-\alpha}\gamma_v}\beta h\rho_v - e^{\gamma_z+\frac{\alpha}{1-\alpha}\gamma_v}h}{\left(e^{\gamma_z+\frac{\alpha}{1-\alpha}\gamma_v}-\beta h\right)\left(e^{\gamma_z+\frac{\alpha}{1-\alpha}\gamma_v}-h\right)}\frac{\alpha}{1-\alpha}\hat{v}_t + \frac{e^{\gamma_z+\frac{\alpha}{1-\alpha}\gamma_v}-\beta h\rho_b}{e^{\gamma_z+\frac{\alpha}{1-\alpha}\gamma_v}-\beta h}\hat{b}_t$$

6. $\hat{\lambda}_t = \hat{R}_t - \rho_z\hat{z}_t - \rho_v\frac{\alpha}{1-\alpha}\hat{v}_t + E_t\{\hat{\lambda}_{t+1} - \hat{\pi}_{t+1}\}$

7. $$\hat{\rho}_t = \left\{\frac{\chi}{\rho}\left[\rho - \bar{B} + \tau\left(\bar{A}-1\right)\right] - \frac{\bar{B}}{\rho}\right\}\hat{u}_t + \frac{\sigma}{\rho}\bar{B}\hat{v}_t + \frac{\bar{B}}{\rho}\left(\hat{\phi}_t - \hat{\lambda}_t\right) - \frac{\tau}{\rho}\left(\bar{A}-1\right)\left(\hat{\varsigma}_t - \hat{\lambda}_t\right) + \frac{\delta}{\rho\left(\delta-\bar{\delta}\right)}\bar{B}\hat{\delta}_t$$

8. $$\hat{\phi}_t = E_t\{\hat{\phi}_{t+1}\} - \rho_z\hat{z}_t - \left[\frac{\alpha}{1-\alpha} + 1 + \beta e^{-\left(\gamma_z+\left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v\right)}\sigma\bar{C}\right]\rho_v\hat{v}_t + \beta e^{-\left(\gamma_z+\left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v\right)}\bar{B}E_t\{\hat{u}_{t+1}\} +$$
$$- \beta e^{-\left(\gamma_z+\left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v\right)}\left(\bar{C} + e^{\sigma\gamma_v}\sigma\bar{\delta}\right)E_t\{\hat{\delta}_{t+1}\} - \left[\bar{D} + \beta e^{-\left(\gamma_z+\left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v\right)}\tau\bar{A}\right]E_t\{\hat{\phi}_{t+1} - \hat{\lambda}_{t+1}\} +$$
$$- \left[\beta e^{-\left(\gamma_z+\left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v\right)}\tau\bar{A} - \frac{\tau}{\rho-\tau}\bar{D}\right]E_t\{\hat{\varsigma}_{t+1} - \hat{\lambda}_{t+1}\} + \frac{\rho}{\rho-\tau}\bar{D}E_t\{\hat{\rho}_{t+1}\}$$

9. $$\hat{\lambda}_t = \hat{\phi}_t + \hat{\mu}_t + S''e^{2\left(\gamma_z+\left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v\right)}\hat{\imath}_{t-1} - (1+\beta)S''e^{2\left(\gamma_z+\left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v\right)}\hat{\imath}_t +$$
$$+ \beta S''e^{2\left(\gamma_z+\left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v\right)}E_t\{\hat{\imath}_{t+1}\} + (\beta\rho_z - 1)S''e^{2\left(\gamma_z+\left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v\right)}\hat{z}_t +$$
$$+ (\beta\rho_v - 1)S''e^{2\left(\gamma_z+\left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v\right)}\left(\frac{\alpha}{1-\alpha}+1\right)\hat{v}_t$$

10. $\hat{k}_t = \hat{u}_t + \hat{\bar{k}}_{t-1} - \hat{z}_t - \left(\frac{\alpha}{1-\alpha}+1\right)\hat{v}_t$

11. $$\hat{\bar{k}}_t = \left(1-\delta e^{\sigma\gamma v}\right) e^{-\left(\gamma_z+\left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v\right)}\hat{\bar{k}}_{t-1} - \delta e^{\sigma\gamma v} e^{-\left(\gamma_z+\left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v\right)}\hat{\delta}_t +$$
$$+\left[1-\left(1-\delta e^{\sigma\gamma_v}\right) e^{-\left(\gamma_z+\left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v\right)}\right]\left(\hat{\mu}_t+\hat{i}_t\right) - \left(1-\delta e^{\sigma\gamma v}\right) e^{-\left(\gamma_z+\left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v\right)}\hat{z}_t +$$
$$-\left[\left(1-\delta e^{\sigma\gamma v}\right)\left(\frac{\alpha}{1-\alpha}+1\right)+\sigma\delta e^{\sigma\gamma v}\right] e^{-\left(\gamma_z+\left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v\right)}\hat{v}_t$$

12. $$\hat{\delta}_t = \frac{\zeta}{\zeta+\bar{\delta}\left(\frac{\tilde{m}}{\bar{k}}e^{\gamma_z+\left(\frac{\alpha}{1-\alpha}+1\right)}\right)^{\sigma}}\left[\sigma\hat{\bar{k}}_{t-1} - \sigma\hat{\tilde{m}}_t - \sigma\hat{d}_t - \sigma\hat{z}_t - \sigma\left(\frac{\alpha}{1-\alpha}+1\right)\hat{v}_t + \eta\hat{u}_t\right]$$

13. $$\hat{\tilde{m}}_t = \tau\left(\frac{\tilde{m}}{\bar{k}}e^{\gamma_z+\left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v}\right)^{-1}\left[\hat{u}_t + \hat{\bar{k}}_{t-1} - \hat{z}_t - \left(\frac{\alpha}{1-\alpha}+1\right)\hat{v}_t\right] - \hat{d}_t$$

14. $$\left[\left(1+\beta\right)e^{2\left(\gamma_z+\frac{\alpha}{1-\alpha}\gamma_v\right)}f'' + \bar{A}\right]\hat{\tilde{m}}_t = e^{2\left(\gamma_z+\frac{\alpha}{1-\alpha}\gamma_v\right)}f''\hat{\tilde{m}}_{t-1} + \beta e^{2\left(\gamma_z+\frac{\alpha}{1-\alpha}\gamma_v\right)}f''E_t\{\hat{\tilde{m}}_{t+1}\} + \bar{A}\hat{\bar{k}}_{t-1} + \bar{A}\frac{\delta}{\delta-\bar{\delta}}\hat{\delta}_t +$$
$$-\left(\bar{A}-1\right)\hat{d}_t - \left(\bar{A}-1\right)\left(\hat{\varsigma}_t - \hat{\lambda}_t\right) + \bar{A}\left(\hat{\phi}_t - \hat{\lambda}_t\right) - \left[\left(1-\beta\rho_z\right)e^{2\left(\gamma_z+\frac{\alpha}{1-\alpha}\gamma_v\right)}f'' + \bar{A}\right]\hat{z}_t +$$
$$-\left[\frac{\alpha}{1-\alpha}\left(1-\beta\rho_v\right)e^{2\left(\gamma_z+\frac{\alpha}{1-\alpha}\gamma_v\right)}f'' + \left(\frac{\alpha}{1-\alpha}+1-\sigma\right)\bar{A}\right]\hat{v}_t$$

15. $$\frac{1}{g}\hat{y}_t - \frac{1}{g}\hat{g}_t = \frac{c}{y}\hat{c}_t + \frac{i}{y}\hat{i}_t + \frac{\tilde{m}}{y}\hat{\tilde{m}}_t + \left\{\rho - \bar{B} + \tau\left[\bar{A}-1\right]\right\}\frac{\bar{k}}{y}e^{-\left(\gamma_z+\left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v\right)}\hat{u}_t$$

16. $$\hat{x}_t = \hat{y} - \left\{\rho - \bar{B} + \tau\left[\bar{A}-1\right]\right\}\frac{\bar{k}}{y}e^{-\left(\gamma_z+\left(\frac{\alpha}{1-\alpha}+1\right)\gamma_v\right)}\hat{u}_t$$

17. $$\hat{g}_{w,t} = \nu\frac{1+\lambda_w}{\lambda_w}\hat{w}_t + \nu\hat{L}_t + \hat{b}_t - \hat{\lambda}_t + \hat{\lambda}_{w,t}$$

18. $$\hat{w}_t = \left(1-\xi_w\right)\hat{\tilde{w}}_t + \xi_w\left(\hat{w}_{t-1} - \hat{\pi}_t - \hat{z}_t - \frac{\alpha}{1-\alpha}\hat{v}_t + \iota_w\hat{\pi}_{t-1} + \iota_w\hat{z}_{t-1} + \iota_w\frac{\alpha}{1-\alpha}\hat{v}_{t-1}\right)$$

19. $$\hat{\tilde{w}}_t = \xi_w\beta E_t\{\hat{\tilde{w}}_{t+1}\} - \xi_w\beta\hat{\pi}_t + \xi_w\beta E_t\{\hat{\pi}_{t+1}\} + \xi_w\beta\left(\rho_z-\iota_w\right)\hat{z}_t + \xi_w\beta\left(\rho_v-\iota_w\right)\frac{\alpha}{1-\alpha}\hat{v}_t + \frac{1-\xi_w\beta}{1+\nu\frac{1+\lambda_w}{\lambda_w}}\hat{g}_{w,t}$$

20. $$\hat{R}_t = \rho_R\hat{R}_{t-1} + \left(1-\rho_r\right)\left[\phi_\pi\hat{\pi}_t + \phi_X\left(\hat{x}_t - \hat{x}_t^*\right)\right] + \phi_{dX}\left[\left(\hat{x}_t - \hat{x}_{t-1}\right) - \left(\hat{x}_t^* - \hat{x}_{t-1}^*\right)\right] + \hat{\eta}_{mp,t}$$

The baseline model is composed of the following 17 linear rational expectations equations in the sticky price-wage economy

1. $\hat{y}_t = \frac{y+F}{y}\alpha\hat{k}_t + \frac{y+F}{y}(1-\alpha)\hat{L}_t$

2. $\hat{\rho}_t = \hat{w}_t - \hat{k}_t + \hat{L}_t$

3. $\hat{s}_t = (1-\alpha)\hat{w}_t + \alpha\hat{\rho}_t$

4. $\hat{\pi}_t = \frac{\iota_p}{1+\beta\iota_p}\hat{\pi}_{t-1} + \frac{\beta}{1+\beta\iota_p}E_t\{\hat{\pi}_{t+1}\} + \frac{(1-\xi_p\beta)(1-\xi_p)}{\xi_p(1+\beta\iota_p)}(\hat{s}_t + \hat{\lambda}_{p,t})$

5. $$\hat{\lambda}_t = \frac{e^{\gamma_z+\frac{\alpha}{1-\alpha}\gamma_v}}{\left(e^{\gamma_z+\frac{\alpha}{1-\alpha}\gamma_v}-\beta h\right)\left(e^{\gamma_z+\frac{\alpha}{1-\alpha}\gamma_v}-h\right)}\hat{c}_{t-1} - \frac{e^{2\left(\gamma_z+\frac{\alpha}{1-\alpha}\gamma_v\right)}+\beta h^2}{\left(e^{\gamma_z+\frac{\alpha}{1-\alpha}\gamma_v}-\beta h\right)\left(e^{\gamma_z+\frac{\alpha}{1-\alpha}\gamma_v}-h\right)}\hat{c}_t +$$

$$+ \frac{e^{\gamma_z+\frac{\alpha}{1-\alpha}\gamma_v}\beta h}{\left(e^{\gamma_z+\frac{\alpha}{1-\alpha}\gamma_v}-\beta h\right)\left(e^{\gamma_z+\frac{\alpha}{1-\alpha}\gamma_v}-h\right)}E_t\{\hat{c}_{t+1}\} + \frac{e^{\gamma_z+\frac{\alpha}{1-\alpha}\gamma_v}\beta h\rho_z - e^{\gamma_z+\frac{\alpha}{1-\alpha}\gamma_v}h}{\left(e^{\gamma_z+\frac{\alpha}{1-\alpha}\gamma_v}-\beta h\right)\left(e^{\gamma_z+\frac{\alpha}{1-\alpha}\gamma_v}-h\right)}\hat{z}_t +$$

$$+ \frac{e^{\gamma_z+\frac{\alpha}{1-\alpha}\gamma_v}\beta h\rho_v - e^{\gamma_z+\frac{\alpha}{1-\alpha}\gamma_v}h}{\left(e^{\gamma_z+\frac{\alpha}{1-\alpha}\gamma_v}-\beta h\right)\left(e^{\gamma_z+\frac{\alpha}{1-\alpha}\gamma_v}-h\right)}\frac{\alpha}{1-\alpha}\hat{v}_t + \frac{e^{\gamma_z+\frac{\alpha}{1-\alpha}\gamma_v}-\beta h\rho_b}{e^{\gamma_z+\frac{\alpha}{1-\alpha}\gamma_v}-\beta h}\hat{b}_t$$

6. $\hat{\lambda}_t = \hat{R}_t - \rho_z\hat{z}_t - \rho_v\frac{\alpha}{1-\alpha}\hat{v}_t + E_t\{\hat{\lambda}_{t+1} - \hat{\pi}_{t+1}\}$

7. $\hat{\rho}_t = \chi\hat{u}_t$

8. $$\hat{\phi}_t = -\rho_z\hat{z}_t - \rho_v(\frac{\alpha}{1-\alpha}+1)\hat{v}_t + e^{-(\gamma_z+(\frac{\alpha}{1-\alpha}+1)\gamma_v)}\beta(1-\delta)E_t\{\hat{\phi}_{t+1}\} +$$

$$+ \left[1 - e^{-(\gamma_z+(\frac{\alpha}{1-\alpha}+1)\gamma_v)}\beta(1-\delta)\right]E_t\{(\hat{\rho}_{t+1} + \hat{\lambda}_{t+1})\}$$

9. $$\hat{\lambda}_t = \hat{\phi}_t + \hat{\mu}_t + e^{2(\gamma_z+(\frac{\alpha}{1-\alpha}+1)\gamma_v)}S''(\beta\rho_z - 1)\hat{z}_t + e^{2(\gamma_z+(\frac{\alpha}{1-\alpha}+1)\gamma_v)}S''(\beta\rho_v - 1)\hat{v}_t +$$

$$- e^{2(\gamma_z+(\frac{\alpha}{1-\alpha}+1)\gamma_v)}S''(\beta+1)\hat{i}_t + e^{2(\gamma_z+(\frac{\alpha}{1-\alpha}+1)\gamma_v)}S''\hat{i}_{t-1} +$$

$$+ e^{2(\gamma_z+(\frac{\alpha}{1-\alpha}+1)\gamma_v)}S''\beta E_t\{\hat{i}_{t+1}\}$$

10. $\hat{\bar{k}}_t = \hat{u}_t + \hat{\bar{k}}_{t-1} - \hat{z}_t - \left(\frac{\alpha}{1-\alpha}+1\right)\hat{v}_t$

11. $\hat{\bar{k}}_t = e^{-(\gamma_z + (\frac{\alpha}{1-\alpha}+1)\gamma_v)}(1-\delta)\left[\hat{\bar{k}}_{t-1} - \hat{z}_t - \left(\frac{\alpha}{1-\alpha}+1\right)\hat{v}_t\right] +$

$$+\left[1 - e^{-(\gamma_z + (\frac{\alpha}{1-\alpha}+1)\gamma_v)}(1-\delta)\right]\left(\hat{i}_t + \hat{\mu}_t\right)$$

12. $\frac{1}{g}\hat{y}_t - \frac{1}{g}\hat{g}_t = \frac{c}{y}\hat{c}_t + \frac{i}{y}\hat{i}_t + \frac{\rho k}{y}\hat{u}_t$

13. $\hat{x}_t = \hat{y}_t - \frac{\rho k}{y}\hat{u}_t$

14. $\hat{g}_{w,t} = \nu\frac{1+\lambda_w}{\lambda_w}\hat{w}_t + \nu\hat{L}_t + \hat{b}_t - \hat{\lambda}_t + \hat{\lambda}_{w,t}$

15. $\hat{w}_t = (1-\xi_w)\hat{\tilde{w}}_t + \xi_w\left(\hat{w}_{t-1} - \hat{\pi}_t - \hat{z}_t - \frac{\alpha}{1-\alpha}\hat{v}_t + \iota_w\hat{\pi}_{t-1} + \iota_w\hat{z}_{t-1} + \iota_w\frac{\alpha}{1-\alpha}\hat{v}_{t-1}\right)$

16. $\hat{\tilde{w}}_t = \xi_w\beta E_t\{\hat{\tilde{w}}_{t+1}\} - \xi_w\beta\hat{\pi}_t + \xi_w\beta E_t\{\hat{\pi}_{t+1}\} + \xi_w\beta(\rho_z - \iota_w)\hat{z}_t + \xi_w\beta(\rho_v - \iota_w)\frac{\alpha}{1-\alpha}\hat{v}_t + \frac{1-\xi_w\beta}{1+\nu\frac{1+\lambda_w}{\lambda_w}}\hat{g}_{w,t}$

17. $\hat{R}_t = \rho_R\hat{R}_{t-1} + (1-\rho_R)\left[\phi_\pi\hat{\pi}_t + \phi_X(\hat{x}_t - \hat{x}_t^\star)\right] + \phi_{dX}\left[(\hat{x}_t - \hat{x}_{t-1}) - (\hat{x}_t^\star - \hat{x}_{t-1}^\star)\right] + \hat{\eta}_{mp,t}$

Note that, in Justiniano et al. (2011), the intertemporal preference shock and the price and wage mark-up shocks are normalized so that they enter the equations of marginal utility of nominal income and the price and wage Phillips curves, respectively, with a unit coefficient. These normalizations, as the authors explain, are convenient for the definition of the priors for the standard deviations of the shocks and for the estimation purposes when the Metropolis-Hasting algorithm is implemented. Normalization requires the definition of new exogenous variables, which are denoted by a 'star', for the three shocks as follows, respectively

$$\hat{b}_t^\star = \frac{(1-\rho_b)\left(e^{\gamma_z+\frac{\alpha}{1-\alpha}\gamma_v} - h\beta\rho_b\right)\left(e^{\gamma_z+\frac{\alpha}{1-\alpha}\gamma_v} - h\right)}{e^{\gamma_z+\frac{\alpha}{1-\alpha}\gamma_v}h + e^{2\left(\gamma_z+\frac{\alpha}{1-\alpha}\gamma_v\right)} + \beta h^2}\hat{b}_t$$

$$\hat{\lambda}_{p,t}^\star = \frac{(1-\xi_p\beta)(1-\xi_p)}{\xi_p(1+\beta\iota_p)}\hat{\lambda}_{p,t}$$

$$\hat{\lambda}_{w,t}^\star = \frac{(1-\beta\xi_w)(1-\xi_w)}{\left(1+\nu\frac{1+\lambda_w}{\lambda_w}\right)(1+\beta)\xi_w}\hat{\lambda}_{w,t}$$

As far as in the maintenance model marginal utility of nominal income, and the price and wage Phillips curves remain unchanged with respect to the baseline model, I implement the normalization procedure of the respective shocks following Justiniano et al. (2011).

F APPENDIX: Data construction

For the construction of the database I have used the series available on the CANSIM database of Canadian Statistics as well as the Bank of Canada. For the estimation exercises of the models I use eight observables, which are

$$\left[\Delta \log X_t \quad \Delta \log C_t \quad \Delta \log \tilde{I}_t \quad \log L_t \quad \Delta \log \frac{W_t}{P_t} \quad \pi_t \quad R_t \quad \Delta \log \frac{P_t^I}{P_t}\right]$$

The above variables are defined as follows

- Output: $\log X_t = \ln(GDP/POPindex) \times 100$
- Consumption: $\log C_t = \ln[(CONS/IMPC)/POPindex] \times 100$
- Investment: $\log \tilde{I}_t = \ln[(INV/IMPC)/POPindex] \times 100$
- Hours: $\log L_t = \ln[(H \times EMPindex/100)/POPindex] \times 100$
- Real wage: $\log W_t/P_t = \ln(COMPH/IMPC) \times 100$
- Inflation: $\pi_t = [\ln(IMPC/IMPC(-1))] \times 100$
- Nominal rate: $R_t = NOMR/4$
- Relative price of investment: $\log(P_t^I/P_t) = \ln(IMPI/IMPC) \times 100$

The terminology on the left-hand side of the above expressions is explained below.

GDP: Real Gross Domestic Product, billions of chained 2007 dollars, seasonally adjusted

$CONS$: Households final consumption expenditure in goods (non-durables and semi-durables) and services, billions of dollars at current prices, seasonally adjusted

INV: Households final consumption expenditure in durable goods plus business gross fixed capital formation, billions of dollars at current prices, seasonally adjusted

$IMPC$: GDP implicit price index (2007Q3=100), average of non-durables, semi-durables and services, seasonally adjusted

$IMPI$: GDP implicit price index (2007Q3=100), average of durables and business gross fixed capital formation, seasonally adjusted

POP: Labor force 15 years and over, number in thousands of civilian, non-institutionalized persons, seasonally adjusted

$POPindex$: POP(2007Q3)=1

EMP: Civilian employment 15 years and over, thousands, seasonally adjusted

$EMPindex$: $100 \times EMP$(2007Q3)=1

H: Average of hours worked (2007Q3=100), business sector, seasonally adjusted

$COMPH$: Total compensation per hour worked (2007Q3=100), business sector, seasonally adjusted

$NOMR$: 3-month treasury bills, percent of seven-day average
Source: Financial market statistics, Bank of Canada

G APPENDIX: Impulse Response Functions

Fig.4 displays the impulse response functions to a positive one standard deviation shock to investment-specific technology process. The IST shock (or disembodied investment specific technology shock) equates, in equilibrium, the inverse of the relative price of investment which, thus, decreases. Following the decline in the relative price of investment the Tobin's q rises on impact of the shock which, combined with the direct effect of the IST shock, generates a decline in real interest despite a rise in the expected marginal product of capital. In the baseline model the strong interest rate effect generates an impact decline in real investment, which depicts an increasing path and overshoots above its steady state value after several periods. The impact decline in real investment induces a fall in new capital and, hence, the level of capital stock falls below its steady state level whereas the cost of installed capital increases. Moreover, the positive investment-specific technology shock generates an immediate decline in effective capital, which keeps on decreasing over the next several periods, despite an impact positive response of the utilization rate, as far as part of installed capital is destroyed. Given the equilibrium path of the effective capital the marginal product of capital rises on impact to the shock as well as over the following periods and reverts its path downward as effective capital starts to recover towards its steady state level. Similarly, capital utilization rate rises above its steady state level on impact of the shock. Give the reduction in the economy's productive capacity output declines on impact. The monetary authority, in order to balance the deviations both of output from its steady state level and of inflation from its target level, reduces the nominal interest rate although by a very small amount. Given the sharp decline in output, the negative wealth effect dominates over the interest rate effect so that agents find it optimal to reduce current consumption on impact of the shock. The price of consumption immediately grows up which in the presence of nominal rigidities pushes downward real wages and increases inflation. As a consequence of the wage and price stickiness both the mark-ups increase and the intermediate goods producers find it optimal to reduce the demand for labor. Hence, in the baseline model output, consumption, investment, hours, real wages, nominal interest rate and effective capital all co-move in response to a positive IST progress.

With regard to the maintenance model the interest rate effect is dominated by the substitution effect. The agents, hence, find it optimal to increase investment on impact of the shock in order to rebuild the destroyed stock of capital. However, the low magnitude in the impact response of real investment and the decline in capital depreciation rate are not sufficient to overhang the negative impact of the IST shock and to push up immediately the amount of capital stock. Moreover, the higher is the elasticity of capital depreciation rate with respect to maintenance the stronger the negative impact of the IST shock on the stock of capital. Therefore, similarly to the baseline model, effective capital declines both on impact of the shock and over the subsequent periods whereas its rental price increases following a hump-shaped path. The impact increase of the capital rental price is additionally boosted by the impact rise in the Tobin's q. As a consequence of a higher marginal product of capital the utilization rate rises on impact following a hump-shaped path. Depreciation rate declines on impact of the IST shock as far as its direct effect dominates the positive effect induced by the increase in utilization. Furthermore, the impact decline in capital stock and its equilibrium dynamics generate a hump-shaped convergence path for the rate of depreciation. Maintenance declines on impact below its steady state level, whereas its relative price increases, and reverts its path over the subsequent periods as the amount of existing capital stock rises and the rate of depreciation accelerates. Differently from the baseline model, the impact response of the nominal interest rate is positive, driven by the strong response of the monetary authority to higher variations in inflation. Output decreases on impact as in the baseline model and overshoots its steady state equilibrium following the sharp peak of investment. Differently from the baseline model, consumption and hours move in opposite directions. Indeed their optimal paths are mirrored through the optimal labor market condition up to the countercyclical effects of the nominal rigidities. So, driven by the negative wealth effect, at optimum, agents decide to recover the reduction in capital by postponing consumption and reducing leisure and thus increasing the hours of work, as well as investment. In the maintenance model, hence, the optimal convergence paths in response to a positive IST shock deliver countercyclical behavior for investment and hours while consumption, depreciation, effective capital and maintenance all co-move with output.

As it can be observed from Fig.4, the impulse response functions of all the considered real variables in the maintenance model depict more hump-shaped paths with respect to those in the baseline model. Moreover, convergence dynamics are slightly faster despite a relatively higher estimated persistency of the IST shock in the maintenance model. This qualitative behavior could be explained by the presence of the maintenance sector which, together with endogenous depreciation rate, serves as an additional transition mechanism for the propagation of the shocks. Maintenance and investment behave as substitutes one to each other in short run. This conclusion supports the findings of Albonico et al. (2014) in which the response functions of the main endogenous variables to a positive IST shock are all pro-cyclical (and positive) with exception of maintenance to capital ratio suggesting, thus, that maintenance and investment are substitutes.

In general, when the IST shock is equated to the inverse of the relative price of investment it is interpreted as the disembodied investment-specific technology progress[28]. This shock aims to reduce the unit cost of production of new investment. In the maintenance model, a positive impact of this shock creates new investment and delivers an increasing optimal path for the capital stock which, given the nature of the shock, is qualitatively comparable to the already existing one. Hence, in the long-run aggregate economy there will be available on the market more capital stock qualitatively equivalent and thus, in aggregate, the optimal lifetime of capital rises.[29] As a consequence of the positive disembodied IST shock, capital depreciation

[28] See, for example, Greenwood et al. (1997) and Boucekkine et al. (2009).

[29] Boucekkine et al. (2009) have shown that the optimal capital lifetime is a decreasing function of the total investment-specific technology progress, both embodied and disembodied, under certain restrictions on the steady state relations of their model parameters. This, in turn, delivers a positive relation between the total IST shock and the capital depreciation rate. In the maintenance model, instead, I distinguish between the

rate decelerates. Differently from the disembodied investment-specific technology progress and according to the literature interpretation the embodied progress increases the marginal productivity of new investment. Hence, I claim that the marginal efficiency of investment technology progress which affects in my model the transformation process of new investment into new capital can reflect the embodied investment shock. In this case, a positive MEI shock aims to improve the quality of the new capital available on the market. The existing capital becomes more obsolete which, thus, accelerates the depreciation path.

The optimal convergence paths to a positive one standard deviation shock to the maintenance-specific technology progress are exhibited in Fig.5. This shock affects the transformation process of the units of final good, used as input by the maintenance-goods producers, into efficiency units of maintenance and is inversely related to the relative price of maintenance. Thus, a positive MST shock reduces the cost of producing one unit of maintenance good on the impact of the shock. Nevertheless, real maintenance declines on impact depicting an increasing path. This occurs, on the one hand, because of a direct negative effect of the MST shock on current real maintenance. On the other hand, the sharp decline both in the price of consumption and in the price of new investment, creates a strong substitution effect which shifts expenditures away from maintenance. Therefore, despite a negative wealth effect generated by an impact decline in real output, households find it optimal to increase consumption and leisure. The rise in Tobin's q makes the real interest rate to fall below its steady state level. Nonetheless, real investment rises on impact as far as the weak interest rate effect is dominated by the substitution effect. Thus, both real consumption and real investment increase on impact and follow a hump-shaped path. Increasing consumption induces households to increase leisure, thus hours worked decline on impact. Marginal utility of labor declines which, in turn, drops real wages below their equilibrium level. The marginal product of capital declines on the impact of the shock due to a strong negative effect of the maintenance relative shadow value, enhanced by an increase in the effective capital. As a consequence, real marginal costs and hence inflation decrease on impact. Given the deviation of inflation from its steady state level and the higher output gap, the monetary authority reduces nominal interest rate. Moreover, the decline in real maintenance accelerates capital depreciation rate which, following a hump-shaped path, destroys current and part of the future capital stock. Moreover, there occurs a one-off event on the capital utilization rate which increases on the impact of the shock due to a temporary positive gap between the economy's aggregate supply and demand. Consequently, effective capital rises above its steady state level. However, it returns to its equilibrium level over the next period as far as the weak effect of increased real investment is contrasted by the increasing path in the depreciation rate and, thus, is not able to boost up enough the amount of capital stock.

two types of the IST progresses which allows me to assert that there exists a positive relation between the (disembodied) IST progress and the capital optimal lifetime and a negative relation between the latter one and the (embodied) MEI progress.

All the considered real variables exhibit same qualitative behavior in response to the labor augmenting technology progress in both the models, see Fig.6. Therefore, all these variables, including depreciation and real maintenance in the maintenance model, co-move with real output. In general, the speed of convergence is slightly faster in the maintenance model with respect to the baseline model, however the difference becomes significant for real investment, capital rental price and utilization rate.

Similarly, as it can be observed in Fig.7, all the real variables including depreciation and maintenance and with exception of the nominal interest rate co-move with real output in response to a positive one standard deviation shock to the monetary policy. Convergence dynamics are significantly different in the optimal responses of investment, wages, effective capital, the marginal product of capital and the utilization rate. On the contrary the magnitude of the impact responses are almost the same.

With regard to the positive government spending shock, Fig.8, consumption and investment move countercyclically with respect to real output. The behavior of this variables is explained by the assumption of a fully Recardian government policy, according to which government finances its spendings issuing short term bonds. Capital depreciation rate decreases on impact of the shock as well, although by a very small amount, and exhibits a well hump-shaped path due to a relatively stronger effect of a decrease in real maintenance with respect to an increase in capital utilization. The convergence dynamics of effective capital, real investment and real wages are significantly faster in the maintenance model as the capital depreciation rate decelerates.

The impulse response functions to a positive one standard deviation intertemporal preference shock, Fig.9, are very similar between the two models. All the real variables are pro-cyclical with exception of investment which decreases as a result of a strong substitution effect towards consumption as well as towards maintenance in the maintenance model. In the latter one depreciation moves countercyclically governed by the optimal equilibrium dynamics of real maintenance.

In response to a positive price mark-up shock, Fig.10, all the real variables are pro-cyclical, including depreciation and maintenance, with exception of inflation and nominal interest rate. An increase in the price mark-up, in fact, rises inflation on impact which, in turn, requires a strong positive interest rate response from the monetary authority. A significant difference in the optimal dynamics between the two models occurs for real consumption. As far as both investment and maintenance decline in the maintenance model, agents are willing to renounce to consumption today more than in the baseline model. In general, convergence is slightly faster in the baseline model despite a relatively higher estimated price friction and almost the same estimated shock persistence.

Finally, with respect to the baseline model, the convergence dynamics in response to a positive wage mark-up shock, Fig.11, result to be more hump-shaped in the maintenance model. Convergence is significantly faster for almost all the real variables. In general, a relatively lower estimated persistence parameter and the presence of an additional adjustment mechanism accelerates the convergence dynamics in the maintenance model. Depreciation and maintenance move in opposite directions, the former one pro-cyclically and the latter one countercyclically. Moreover, differently from the baseline model, real investment in the maintenance model is pro-cyclical. In fact, in the baseline model the slight increase in real investment is explained by the substitution effect which moves away from consumption. In the maintenance model, instead, the increased demand for maintenance followed by a rise in the stock of capital tears both real consumption and investment down below their steady state levels.

Fig. 4: Impulse response functions to one standard deviation shock to IST

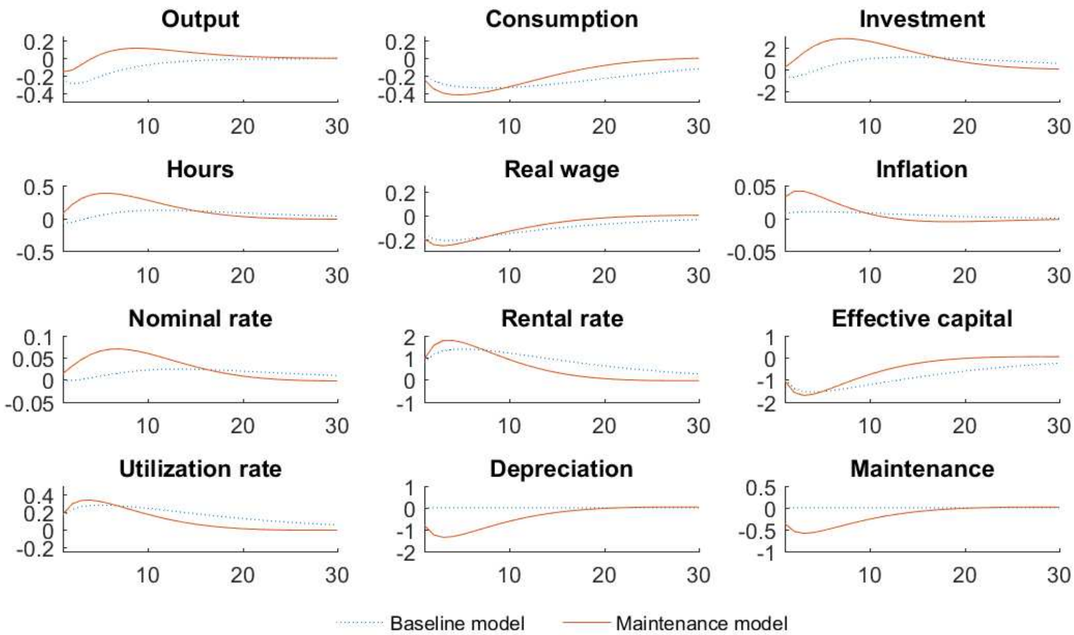


Fig. 5: Impulse response functions to one standard deviation shock to MST

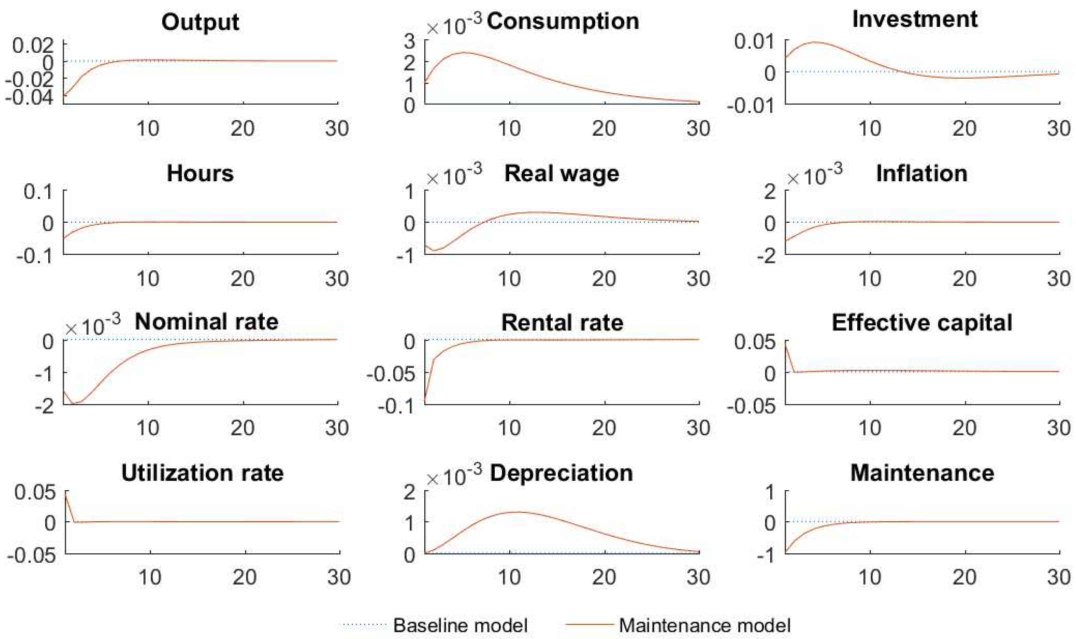


Fig. 6: Impulse response functions to one standard deviation shock to labor augmenting technology

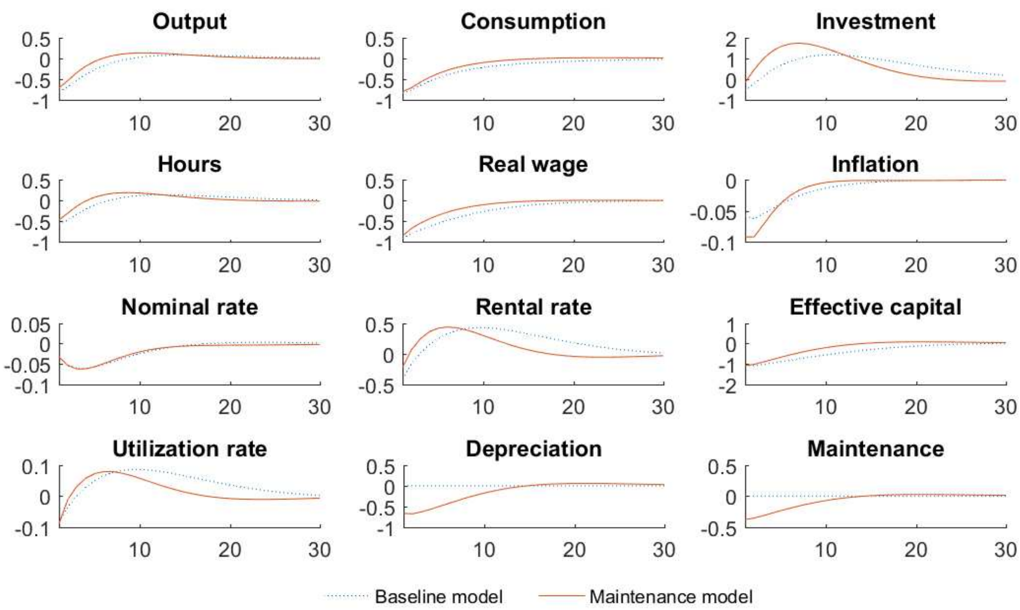


Fig. 7: Impulse response functions to one standard deviation shock to monetary policy

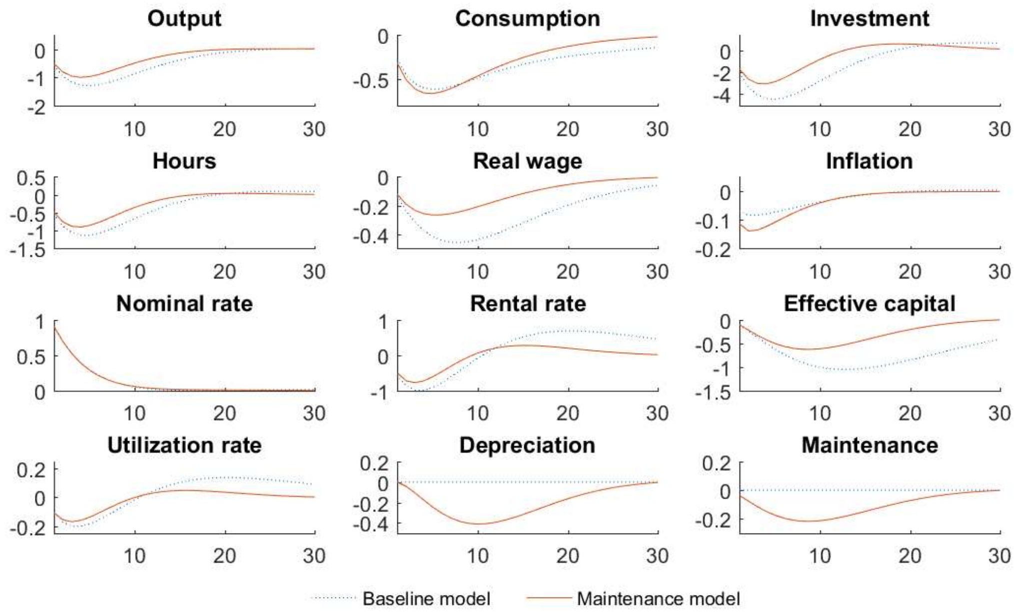


Fig. 8: Impulse response functions to one standard deviation shock to government spending

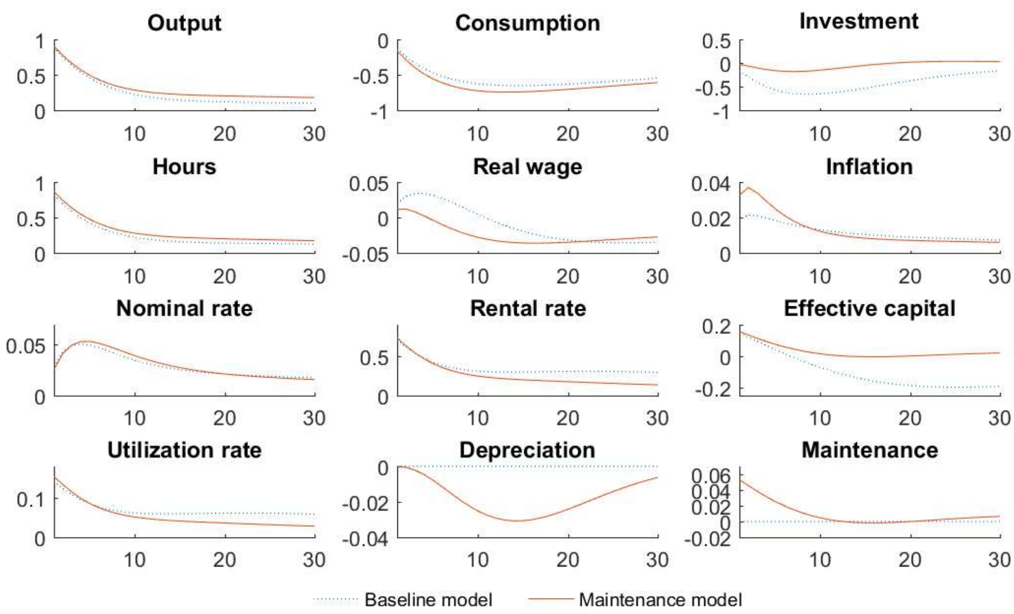


Fig. 9: Impulse response functions to one standard deviation shock to intertemporal preferences

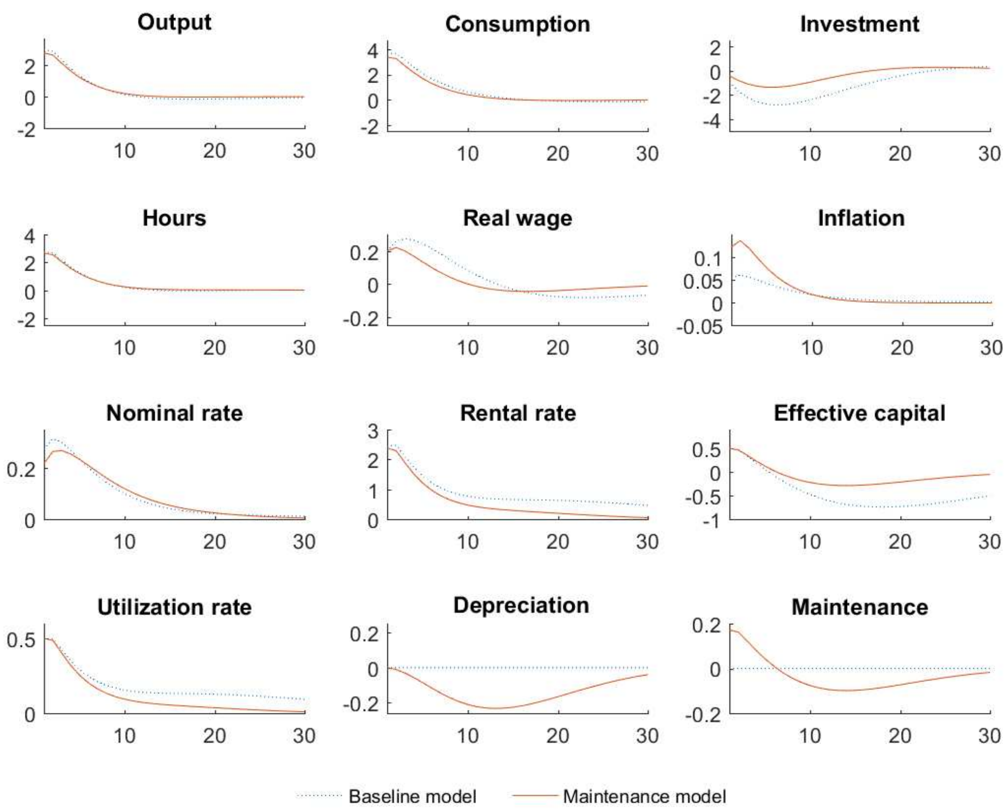


Fig. 10: Impulse response functions to one standard deviation shock to price mark-up

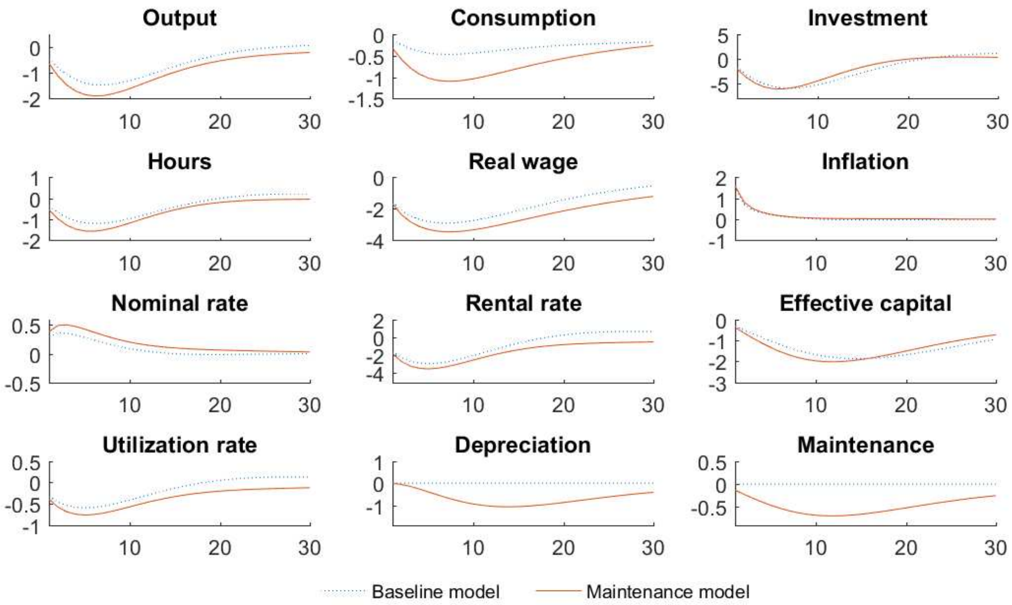


Fig. 11: Impulse response functions to one standard deviation shock to wage mark-up